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A Model for the Statistics of Concentration Fluctuations Arising from Dual Source Releases

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A MODEL FOR THE STATISTICS OF CONCENTRATION FLUCTUATIONS ARISING FROM DUAL SOURCE RELEASES

by

Eugene Yee

PCN No. 6QD11

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ABSTRACT

The higher-order correlation functions for the concentration fluctuations arising from a two-point-source configuration have been calculated analytically within the context of the phenomenology of a fluctuating plume model (viz., a meandering plume model which explicitly incorporates internal fluctuations). Explicit expressions for the second-, third-, and fourth-order correlation functions between the concentration fluctuations produced by two point sources are given in terms of the source separation d and the 5 physically based parameters that define the generalized fluctuating plume model: namely, the absolute plume dispersion, σ_a, which determines the outer plume length scale; the relative plume dispersion, σ_r, which determines the inner plume length scale; the fluctuation intensity, i_r, in relative coordinates which determines the internal concentration fluctuation level; the correlation coefficient, r, between the positions of the centroids of the two interfering plumes; and, the correlation coefficient, r^*, between the concentration fluctuations of the two plumes in relative coordinates which determines the degree of internal mixing of the two scalars. Results for the profiles of the second-order correlation function, ρ, have been compared with some new experimental data for a two-point-source configuration in grid turbulence generated in a water channel simulation. These results are in good agreement with the data and suggests that the analytical model for ρ can reproduce many qualitative trends.

Although the correlation coefficient, r^*, between plume concentrations in a relative coordinate system was found to be approximately constant over the range of streamwise measurements made in the water channel, it seems reasonable to expect that r^* in general would be dependent on x. This results from the fact that r^* is a measure of the degree of mixing between two scalars when they come into contact, and this degree of mixing must evolve with downstream distance. Indeed, for two scalars with equal molecular diffusivities, it is expected that, with continued mixing with increasing x, the fluctuation intensity i_r in the relative coordinate system will decay to zero and r^* will increase to unity in the far field as the two scalars become completely mixed with each other (and, hence, perfectly correlated) in the final, fully-mixed asymptotic state characterizing the turbulent diffusive regime of plume development. The problem of mixing between two scalars becomes more complex, however, in the case of unequal molecular diffusivities. In this case, the phenomenon of differential diffusion can cause the two scalars to become decorrelated in the far field (i.e., $r^* \rightarrow 0$), although this molecular effect is expected to weaken with increasing Reynolds number and, hence, may not be a problem for high-Reynolds-number turbulence (such as that which exists in the atmospheric boundary-layer). In consequence, to represent the behavior of the higher-order correlation functions over the whole streamwise domain will require the specification of an evolution (transport) equation for the parameter r^*. It is expected that future work will concentrate on this aspect of the modelling. Furthermore, extension of the model to include more than two sources will be undertaken.

Finally, detailed measurements of the higher-order correlation between concentration fluctuations of two sources are presently not available. To verify (either qualitatively or quantitatively) the model features predicted to be present in the higher-order correlation will require some new measurements of passive scalar dispersion from two separated sources. This work highlights the urgent need for thorough experimentation in this area.

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I. INTRODUCTION

The dispersion of a passive scalar in the turbulent atmospheric boundary-layer is a phenomenon of both practical and fundamental interest. In consequence, a number of observational programs for the measurement of concentration fluctuations in dispersing plumes have been undertaken in recent years with the objectives of improving our understanding of the detailed plume concentration structure and of developing improved models for the statistical description of concentration fluctuations. These experimental investigations have included both laboratory and full-scale field studies. Fackrell and Robins [1] and Bara et al. [2] studied point source dispersion in a turbulent, neutrally stratified boundary-layer in a wind tunnel and water channel, respectively. Deardorff and Willis [3] made detailed measurements of concentration fluctuations in buoyant and non-buoyant plumes under convective conditions in a water tank. Stapountzis et al. [4] studied line source heat dispersion in grid turbulence. In conjunction with these studies, a number of full-scale atmospheric measurements of concentration fluctuations from a continuous point source at or near the ground have been made under a variety of meteorological conditions ranging from convective to extremely stable (e.g., Hanna [5]; Dinar et al. [6]; Mylne and Mason [7]; Mylne [8]; Yee et al. [9], [10]).

Although there are now comprehensive datasets documenting the nature of concentration fluctuations in plumes from a single source release, there is still a need for similar measurements to be made of the joint concentration statistics in overlapping plumes arising from the release of material injected into the flow at two or more separate locations. Unfortunately, there has been relatively little effort expended in the measurement and investigation of the interference of passive scalar fields produced by two or more sources. In this regard, notable exceptions are afforded by several experimental investigations of the dispersion and mixing arising from the partial overlap of passive plumes released from two separated sources: Warhaft [11] investigated concentration statistics arising from the superposition of two passive thermal wakes downstream of two parallel continuous cross-stream line sources in grid turbulence; and, Sawford et al. [12] conducted atmospheric dual source tracer experiments using two tracers, sulfur hexafluoride (SF_6) and phosphorous smoke (P), to identify the contribution of the two sources to the total concentration field measured at a given receptor point. More recently, some field experiments on the

interaction of plumes from two tracer sources (ammonia and propane) in the atmospheric surface layer have been undertaken by Davis et al. [25]. Motivated by some of these (albeit limited) experimental measurements, Kaplan and Dinar [13] and Thomson [14] have formulated various two-particle dispersion models to calculate the second-order correlation between the concentration fluctuations resulting from two separated sources. These models, which involve the direct simulation of the trajectories of fluid particle pairs, can provide the second-order moment structure of the concentration field (e.g., concentration variance, second-order correlation between the concentration resulting from two separated sources), but their computational cost is high. Furthermore, stochastic models for the motion of particle pairs cannot provide any information on the higher-order moment structure of the concentration field (e.g , higher concentration moments, including shape parameters such as skewness and kurtosis, and higher-order correlations of concentration fluctuations produced by two separated sources).

The aim of this report is to develop an analytical model for the joint statistics between the concentration resulting from two separated continuous point sources. This model is based on a generalization of the fluctuating plume approach introduced by the author (Yee and Wilson [15]; Yee et al. [16]) which explicitly incorporates the effects of internal fluctuations in a meandering plume model. The development of this analytical model allows closed-form expressions to be obtained for all the one-point joint concentration statistics arising from the partial overlap and interference of passive scalars released from two separated continuous point sources. In its present state of development, it is hoped that this model can serve as a metaphor or analogy for the modelling of the joint statistics (e.g., higher-order correlation of concentration) between scalars emanating from a pair of point sources. Despite certain idealizations, it will be shown that this analytical model does appear to exhibit correct qualitative behavior for the second-order correlation. Furthermore, use of a simple analytical model helps understanding of the behavior and features to be expected in the higher-order correlations between the concentrations from two separated sources. In particular, this model will be used as a framework for the analysis and interpretation of the observed trends in some new measurements of the second-order correlation between the saline concentrations resulting from two separated point sources in grid turbulence generated in a water channel simulation.

II. MODEL FORMULATION

II.1. Model for an isolated source

The minimal requirement for a self-contained picture of the expected concentration fluctuation levels in a dispersing plume is a set of laws or models governing the phenomenological aspects of external and internal fluctuations that summarize the effects of the different relevant length scales of the turbulence and the developing plume on both the dispersion and mixing processes. Scales of turbulence that are much larger than the characteristic diameter (mean instantaneous width) of a plume will simply result in the irregular bulk meandering of the plume giving rise to the external fluctuations. Scales of turbulence comparable to or smaller than the mean instantaneous plume width are responsible for turbulent stirring and molecular mixing inside the instantaneous plume boundaries and give rise to the internal fluctuations.

The fluctuating plume approach to the prediction of concentration fluctuation statistics for an isolated point source plume dispersing in homogeneous isotropic turbulence with a uniform mean flow is a natural extension of Gifford's [17] meandering plume model that accounts only for the effects of bulk meandering of the plume (external concentration fluctuations). For the isolated point source, it is convenient to introduce a right-handed Cartesian coordinate system with the point source as the origin. In this coordinate system, x denotes the streamwise distance from the source in the direction of the mean flow, and $\mathbf{x}_T \equiv (y, z)$ is the projection of the position vector $\mathbf{x}$ on the transverse y-z plane. Because the dispersion is assumed to be isotropic in the transverse plane, it is evident that the statistics of the plume concentration field in the transverse plane can depend only on distance from the origin in this plane (viz., the statistics at any fixed downstream transverse plane can depend only on $y^2 + z^2$ and, hence, are rotationally invariant in this plane). The fluctuating plume model which incorporates internal fluctuations in a meandering plume model is encapsulated in the following fundamental relation (Yee and Wilson [15]):

$$\begin{aligned} f\big(\chi;(x,\mathbf{x}_T)\big) &= \int f_r\big(\chi;(x,\mathbf{x}_{T,r})\big) f_c\big(\mathbf{x}_{T,c};x\big)\, d\mathbf{x}_{T,c} \\ &= \int f_r\big(\chi;(x,\mathbf{x}_T-\mathbf{x}_{T,c})\big) f_c\big(\mathbf{x}_{T,c};x\big)\, d\mathbf{x}_{T,c}, \end{aligned} \tag{1}$$

where the integration is over the entire transverse plane of instantaneous plume centroid positions at the given fixed downstream distance, x, from the source. In Equation (1),

χ are the possible values that can be assumed by the instantaneous plume concentration $\phi(\mathbf{x})$ at location $\mathbf{x}$: $\mathbf{x}_{T,c} \equiv (y_c, z_c)$ is the position of the instantaneous plume centroid in a transverse plane at a fixed downstream distance, x, from the source; and, $\mathbf{x}_{T,r} \equiv (y_r, z_r)$ are the transverse coordinates of a point measured relative to the position $\mathbf{x}_{T,c}$ of the instantaneous plume centroid. In the second line of Equation (1), the relationship $\mathbf{x}_T = \mathbf{x}_{T,r} + \mathbf{x}_{T,c}$ has been used. In words, Equation (1) states that the concentration probability density function (PDF), $f(\chi;(x,\mathbf{x}_T))$, in the absolute (fixed) reference frame is determined as the convolution transform (Hirschman and Widder [26]) of the concentration PDF, $f_r(\chi;(x,\mathbf{x}_{T,r}))$, in the relative (moving) reference frame attached to the instantaneous plume centroid, with the kernel of the transform being the spatial PDF, $f_c(\mathbf{x}_{T,c};x)$, of the instantaneous plume centroid position.

For isotropic dispersion, it has been shown by Yee and Wilson [15] that the spatial position PDF, f_c, of the plume centroid is uniquely determined as a consequence of two geometrical invariance properties which lead to the following isotropic Gaussian form:

$$f_c(y_c, z_c; x) = \frac{1}{2\pi\sigma_m^2(x)} \exp\left(-\frac{y_c^2 + z_c^2}{2\sigma_m^2(x)}\right), \tag{2}$$

where $\sigma_m(x)$ is the standard deviation (spread) of the plume centroid position in the transverse plane at the downstream distance, x. Furthermore, it appears that the relative concentration PDF, f_r, can be represented to a reasonable approximation by a gamma PDF with the form (Yee et al. [16]; Yee and Wilson [15])

$$f_r(\chi;(x,\mathbf{x}_{T,r})) = \frac{k^k}{C_r\Gamma(k)} \left(\frac{\chi}{C_r}\right)^{k-1} \exp\left(-\frac{k\chi}{C_r}\right), \tag{3}$$

where C_r is the ensemble-mean concentration of the dispersing plume determined with respect to the plume centroid, and $k \equiv 1/i_r^2$ with i_r the concentration fluctuation intensity (i.e., standard deviation to mean concentration ratio) determined in the relative reference frame.

Since f_c and f_r determine f in accordance to Equation (1), they also determine all the moments of concentration. To calculate these concentration moments, we need to specify a functional form for mean concentration C_r [cf. Equation (3)]. In view of the isotropic dispersion, a reasonable approximation for C_r is the circular symmetric Gaussian form given by

$$C_r(x,\mathbf{x}_{T,r}) = C_{r,0}(x) \exp\left(-\frac{\mathbf{x}_{T,r}\cdot\mathbf{x}_{T,r}}{2\sigma_r^2(x)}\right), \tag{4}$$

where σ_r is the relative plume dispersion and $C_{r,0}(x)$ is the maximum (centerline) value of C_r for the plume cross-section at downstream distance x. Now, substituting f_c and f_r from Equations (2) and (3), respectively, into Equation (1), applying the approximation for C_r in Equation (4), and using the definition for the n-th concentration moment about the origin

$$\langle \phi^n(\mathbf{x}) \rangle \equiv \int_0^\infty \chi^n f(\chi; \mathbf{x})\, d\chi, \tag{5}$$

it is readily shown that (Yee and Wilson [15])

$$\frac{\langle \phi^n(\mathbf{x}) \rangle}{C_{r,0}^n(x)} = \frac{1}{k^n} \frac{\Gamma(n+k)}{\Gamma(k)} \frac{1}{\left[1 + n\left(\frac{\sigma_m}{\sigma_r}\right)^2\right]} \exp\left(- \frac{y^2 + z^2}{2\left(\frac{\sigma_r^2}{n}\right)\left[1 + \left(\frac{\sigma_m}{\sigma_r}\right)^2\right]} \right). \tag{6}$$

In Equation (6), the operation $\langle \cdot \rangle$ denotes ensemble averaging, $\Gamma(x)$ is the gamma function, and the dependence of σ_m, σ_r, and k on x has been suppressed for notational simplicity. Finally, it is noted that Equation (6) with $n = 1$ implies that the mean plume dispersion, σ_a, the relative plume dispersion, σ_r, and the plume centroid dispersion, σ_m, are related by

$$\sigma_a^2(x) \equiv \sigma_r^2(x)\big(1 + M(x)\big) = \sigma_r^2(x) + \sigma_m^2(x), \tag{7}$$

where $M(x) \equiv \sigma_m^2(x)/\sigma_r^2(x)$ is the meander ratio.

II.2. Model for two separated sources

The concentration statistics arising from an isolated point source in isotropic turbulence are rotationally invariant in the transverse plane [cf. Equation (6)]. Introducing another point source into this turbulent flow selects out one direction and clearly explicitly breaks the rotational invariance. Without any fundamental loss in generality, it is assumed that the two sources are located at $(x, y) = (0, 0)$ and are separated by a distance d along the z-axis, with the origin of the right-handed Cartesian coordinate system lying midway between the sources. To be more specific, it is assumed that sources 1 and 2 are located at $(x, y, z) = (0, 0, -d/2)$ and $(0, 0, d/2)$, respectively. It is assumed that sources 1 and 2 are identical except for their location.

As the two plumes emanating from sources 1 and 2 come into contact, partially overlap, and mix, the joint statistics of the two concentration fields at a given receptor

point are embodied in the one-point joint concentration PDF, $f(\chi_1, \chi_2; \mathbf{x})$, in the "absolute" reference frame defined by

$$f(\chi_1, \chi_2; \mathbf{x}) d\chi_1 \, d\chi_2 = \Pr\left\{\chi_1 \le \phi_1(\mathbf{x}) < \chi_1 + d\chi_1, \chi_2 \le \phi_2(\mathbf{x}) < \chi_2 + d\chi_2\right\}, \qquad (8)$$

where $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$ are the concentrations contributed by sources 1 and 2, respectively, at the position $\mathbf{x}$. This joint concentration PDF explicitly incorporates the effects of meandering and internal fluctuations from both plumes in their contribution to the determination of the inter-dependence of the concentrations made from each individual source. The fundamental relation summarized in Equation (1) for an isolated source is generalized to the following relationship for two separated sources:

$$f(\chi_1, \chi_2; (x, \mathbf{x}_T)) = \iint f_r(\chi_1, \chi_2; (x, \mathbf{x}_T - \mathbf{x}_{T,c}^{(1)}), (x, \mathbf{x}_T - \mathbf{x}_{T,c}^{(2)})) \times f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)}; x) \, d\mathbf{x}_{T,c}^{(1)} \, d\mathbf{x}_{T,c}^{(2)}, \qquad (9)$$

where the integration is over the entire transverse planes for $\mathbf{x}_{T,c}^{(1)}$ and $\mathbf{x}_{T,c}^{(2)}$ at x. In Equation (9), $(\mathbf{x}_T - \mathbf{x}_{T,c}^{(j)}) \equiv \mathbf{x}_{T,r}^{(j)}$ $(j = 1, 2)$ denotes the transverse coordinates of a point measured relative to the position of the instantaneous plume centroid of the plume released from source j.

We can regard $f_r(\chi_1, \chi_2; (x, \mathbf{x}_{T,r}^{(1)}), (x, \mathbf{x}_{T,r}^{(2)}))$ as representing the two-point joint concentration PDF in the "relative" frame. Furthermore, $f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)}; x)$ in Equation (9) is the joint spatial PDF of the positions of the instantaneous centroids of the two plumes in a transverse plane at the downstream distance x. The two-point joint PDF of concentration, $f_r(\chi_1, \chi_2; (x, \mathbf{x}_{T,r}^{(1)}), (x, \mathbf{x}_{T,r}^{(2)}))$, in the "relative" frame is simply interpreted as the concentration PDF that would result in the absence of any meandering motions in the two plumes (viz., this joint concentration PDF only incorporates the effects of internal fluctuations from the two plumes on their interaction). In this case, the instantaneous centroid positions of the two plumes are assumed to be fixed in space with the result that $f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)}; x) = \delta(\mathbf{x}_{T,c}^{(1)} - \mathbf{x}_{c,1}(x))\delta(\mathbf{x}_{T,c}^{(2)} - \mathbf{x}_{c,2}(x))$, where $\mathbf{x}_{c,1}(x)$ and $\mathbf{x}_{c,2}(x)$ are the fixed centroid positions of plumes 1 and 2, respectively, at downstream distance x and $\delta(\cdots)$ denotes the Dirac delta function. From this perspective then, $f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)}; x)$ embodies the effects of the plume meander on the correlations between the concentrations from the two laterally separated sources. Hence, Equation (9) separates the contributions of meandering and internal fluctuations to the interaction of the plumes from the two individual sources.

To use Equation (9) to predict the joint statistics of concentration resulting from the pair of sources, we need to specify functional forms for $f_r(\chi_1, \chi_2)$ (suppressing the dependence on x, $\mathbf{x}_{T,r}^{(1)}$, and $\mathbf{x}_{T,r}^{(2)}$ for notational simplicity) and $f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)})$ (suppressing the dependence on x). To generalize Equation (2) for the spatial PDF of the plume centroid location for a single source, it is natural to assume that the positions of the instantaneous centroids of the two plumes resulting from two sources are jointly Gaussian (which is certainly a reasonable approximation) with the following form:

$$f_c(\mathbf{x}_{T,c}^{(1)}, \mathbf{x}_{T,c}^{(2)}) = \frac{1}{2\pi\sigma_m^2(1-r^2)^{1/2}} \exp\left(-\frac{\left(y_c^{(1)^2} - 2r y_c^{(1)} y_c^{(2)} + y_c^{(2)^2}\right)}{2(1-r^2)\sigma_m^2}\right)$$
$$\times \frac{1}{2\pi\sigma_m^2(1-r^2)^{1/2}} \exp\left(-\frac{\left(z_c^{(1)^2} - 2r z_c^{(1)} z_c^{(2)} + z_c^{(2)^2}\right)}{2(1-r^2)\sigma_m^2}\right). \qquad (10)$$

Here, $\mathbf{x}_{T,c}^{(j)} \equiv (y_c^{(j)}, z_c^{(j)})$ $(j = 1, 2)$ and the assumptions involved in Equation (10) are that the y-centroid position is independent of the z-centroid position in isotropic dispersion and that $(y_c^{(1)}, y_c^{(2)})$ and $(z_c^{(1)}, z_c^{(2)})$ are each jointly normally distributed with a correlation coefficient r. It is noted that both σ_m and r in Equation (10) have an explicit dependence on x.

We must also postulate a form for $f_r(\chi_1, \chi_2)$. In conjunction with Equation (3), it seems natural to assume that the joint concentration PDF in the relative frame can be reasonably approximated using the bivariate gamma distribution:

$$f_r(\chi_1, \chi_2) = \frac{1}{C_{r,1}C_{r,2}(1-r^*)} \frac{k^{k+1}}{\Gamma(k)} \left(\frac{\chi_1\chi_2}{C_{r,1}C_{r,2}r^*}\right)^{(k-1)/2}$$
$$\times \exp\left(-\frac{k}{(1-r^*)}\left[\frac{\chi_1}{C_{r,1}} + \frac{\chi_2}{C_{r,2}}\right]\right) I_{k-1}\left(\frac{2k}{(1-r^*)}\sqrt{\frac{\chi_1\chi_2}{C_{r,1}C_{r,2}}r^*}\right), \qquad (11)$$

where $C_{r,1}$ and $C_{r,2}$ denote the ensemble-mean concentration of the plume dispersing from sources 1 and 2, respectively, measured relative to their respective instantaneous centroids; k is the inverse of the square of the fluctuation intensity, i_r^2, in the relative frame as defined earlier for the isolated source; r^* is the correlation coefficient between the two concentrations (due to the mixing of the two scalars); and, $I_n(\cdot)$ is a modified Bessel function of the first kind of order n (Spanier and Oldham [18]). No satisfying *a priori* justification can be offered for applying the functional form of Equation (11) for f_r. However, a motivation for the use of this functional form arises from the fact that its marginal distributions reduce

to a gamma PDF for the concentration in relative coordinates of each source. The use of a gamma PDF for the concentration in relative coordinates resulting from a single source can be justified *a posteriori* from the performance of the fluctuating plume model in a number of different situations (e.g., Yee and Wilson [15]; Yee et al. [16]).

In Equation (11), it is assumed that k and r^* depend on x only. This implies that the fluctuation intensity and concentration correlation coefficient in the relative frame, arising from the small-scale (in-plume) mixing processes, are constant in a transverse plane through the overlapping plumes at any fixed downstream distance, x. Furthermore, in the absence of bulk meandering, when the two scalars are completely mixed with each other, the scalars become perfectly correlated and $r^* = 1$. In view of Equation (4) and the assumption that the two sources are displaced $\pm d/2$ from the origin along the z-axis, it follows that the mean concentration distributions, $C_{r,j}$ $(j = 1, 2)$, in relative coordinates have the following forms:

$$C_{r,1}(x, \mathbf{x}_{r,r}^{(1)}) = C_{r,0}(x) \exp\left(-\frac{\left(y - y_c^{(1)}\right)^2 + \left(z - z_c^{(1)} + d/2\right)^2}{2\sigma_r^2(x)} \right), \tag{12a}$$

and

$$C_{r,2}(x, \mathbf{x}_{r,r}^{(2)}) = C_{r,0}(x) \exp\left(-\frac{\left(y - y_c^{(2)}\right)^2 + \left(z - z_c^{(2)} - d/2\right)^2}{2\sigma_r^2(x)} \right), \tag{12b}$$

where the maximum centerline value $C_{r,0}$ for each plume is assumed to be equal (viz., the release rate, Q, is assumed to be the same for each source).

With the approximations for f_c and f_r in Equations (10) and (11), the one-point joint concentration PDF, f, in the absolute frame is uniquely determined in accordance to the fundamental relation of Equation (9). As a consequence, the joint concentration statistics in terms of those from the individual sources can be determined explicitly. In particular, if $R(\phi_1, \phi_2)$ is a function of the individual concentrations $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$ at a given location $\mathbf{x}$, the unconditional expectation of R can then be obtained by

$$\left\langle R\left(\phi_1(\mathbf{x}), \phi_2(\mathbf{x})\right)\right\rangle = \iint_0^\infty R(\chi_1, \chi_2) f(\chi_1, \chi_2; \mathbf{x})\, d\chi_1\, d\chi_2. \tag{13}$$

In particular, the choice $R(\phi_1, \phi_2) = \phi_1^l \phi_2^m$ yields the joint moments of the two concentration fields at a given point $\mathbf{x}$ of orders l and m. If we substitute this form for R in Equation (13) and utilize the fundamental relation of Equation (9), the problem of calculating the one-point joint concentration moment reduces to the evaluation of the following

multiple integral:

$$\langle \phi_1^l(\mathbf{x})\phi_2^m(\mathbf{x})\rangle = \iiiint \chi_1^l \chi_2^m f_r(\chi_1,\chi_2;(x,\mathbf{x}_T-\mathbf{x}_{T,c}^{(1)}),(x,\mathbf{x}_T-\mathbf{x}_{T,c}^{(2)})) \times f_c(\mathbf{x}_{T,c}^{(1)},\mathbf{x}_{T,c}^{(2)};x)\,d\mathbf{x}_{T,c}^{(1)}\,d\mathbf{x}_{T,c}^{(2)}\,d\chi_1\,d\chi_2. \quad (14)$$

This multiple integral can be evaluated by first doing the integral over χ_1 and χ_2 with the functional form for f_r given by the bivariate gamma distribution of Equation (11). This yields

$$\iint_0^\infty d\chi_1\,d\chi_2\chi_1^l\chi_2^m f_r(\chi_1,\chi_2) = \frac{\Gamma(k+l)\Gamma(k+m)}{k^{l+m}\Gamma^2(k)} \times {}_2F_1(-l,-m;k;r^*)C_{r,1}^l(x,\mathbf{x}_{T,r}^{(1)})C_{r,2}^m(x,\mathbf{x}_{T,r}^{(2)}), \quad (15)$$

where ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function with numeratorial parameters a and b, denominatorial parameter c, and argument x (Spanier and Oldham [18]). Next, substituting Equations (15) and (10) into Equation (14), and using Equations (12a) and (12b) for the forms of the ensemble-mean concentration distributions in the relative frame, it is seen that the remaining integral has an integrand that is simply a product of multivariate Gaussian distributions. It is noted that this final operation, up to a constant factor, simply corresponds to the integration of the product of $C_{r,1}^l$ and $C_{r,2}^m$ over the joint PDF of centroid displacements $\mathbf{x}_{T,c}^{(1)}$ and $\mathbf{x}_{T,c}^{(2)}$ of the plumes produced by the two-point-source configuration. This is a standard integral that is readily evaluated by completing squares with the objective of expressing the arguments of the exponentials as quadratic forms. Following this procedure, the resulting integrals can now be evaluated straightforwardly to yield the following explicit expression for the one-point joint concentration moment:

$$\frac{\langle \phi_1^l(\mathbf{x})\phi_2^m(\mathbf{x})\rangle}{C_{r,0}^{l+m}(x)} = \frac{\Gamma(k+l)\Gamma(k+m)}{k^{l+m}\Gamma^2(k)}\,{}_2F_1(-l,-m;k;r^*)\frac{1}{D^{[l,m]}} \times \exp\left(-\frac{1}{2}\frac{(y^2+z^2)}{\sigma_r^2/(l+m)}\left[1-\frac{l^2m^2}{(l+m)}\frac{(1-r^2)^2M^4}{(D^{[l,m]})^2}\mathcal{G}_{l|m}^{(1)}(r,M)\right]\right) \times \exp\left(-\frac{1}{2}\frac{zd}{\sigma_r^2/(m-l)}\left[1-\frac{l^2m^2}{(m-l)}\frac{(1-r^2)^2M^4}{(D^{[l,m]})^2}\mathcal{G}_{l|m}^{(2)}(r,M)\right]\right) \times \exp\left(-\frac{1}{2}\frac{d^2/4}{\sigma_r^2/(l+m)}\left[1-\frac{l^2m^2}{(l+m)}\frac{(1-r^2)^2M^4}{(D^{[l,m]})^2}\mathcal{G}_{l|m}^{(3)}(r,M)\right]\right), \quad (16)$$

where

$$D^{[l,m]} \equiv 1+(l+m)M+(1-r^2)lmM^2; \quad (17a)$$

$$\mathcal{G}^{(1)}_{l|m}(r,M) \equiv lA_{[l]}(A_{[m]}+B_{[l]})^2 - \frac{2r}{(1-r^2)M^2}(A_{[m]}+B_{[l]})(A_{[l]}+B_{[m]}) + mA_{[m]}(A_{[l]}+B_{[m]})^2; \tag{17b}$$

$$\mathcal{G}^{(2)}_{l|m}(r,M) \equiv lA_{[l]}(B^2_{[l]}-A^2_{[m]}) - \frac{2r}{(1-r^2)M^2}(A_{[l]}B_{[l]}-A_{[m]}B_{[m]}) + mA_{[m]}(A^2_{[l]}-B^2_{[m]}); \tag{17c}$$

and,

$$\mathcal{G}^{(3)}_{l|m}(r,M) \equiv lA_{[l]}(A_{[m]}-B_{[l]})^2 + \frac{2r}{(1-r^2)M^2}(A_{[m]}-B_{[l]})(A_{[l]}-B_{[\tilde{m}]}) + mA_{[m]}(A_{[l]}-B_{[m]})^2 \tag{17d}$$

In Equations (17a) to (17d), $A_{[n]}$ and $B_{[n]}$ are functions of r and M that are defined as follows:

$$A_{[n]} \equiv 1 + \frac{1}{n}\frac{1}{(1-r^2)M}, \tag{18a}$$

and

$$B_{[n]} \equiv \frac{1}{n}\frac{r}{(1-r^2)M}. \tag{18b}$$

Note that $\mathcal{G}^{(1)}_{l|m}$ and $\mathcal{G}^{(3)}_{l|m}$ are symmetric and $\mathcal{G}^{(2)}_{l|m}$ is anti-symmetric under exchange of l and m. In consequence, the joint concentration moments with $l = m$ are rotationally invariant in the transverse plane because $\mathcal{G}^{(2)}_{l|m} = 0$ and the symmetry-breaking term involving zd in Equation (16) vanishes identically.

The predictions of the one-point joint concentration moments obtained from the generalized fluctuating plume model require as input the following 5 parameters, each of which are functions of downstream distance, x; namely, (1) the mean plume dispersion (or, outer plume length scale) σ_a; (2) the relative plume dispersion (or, inner plume length scale) σ_r; (3) the relative in-plume fluctuation measure $k \equiv 1/\imath_r^2$, (4) the correlation coefficient, r, between the positions of the two instantaneous plume centroids; and, (5) the correlation coefficient, r^*, between concentrations from the two separated sources due only to in-plume mixing processes (viz., without any influence from the bulk meandering of the two plumes). The evolution of these parameters with downstream distance x will be a function of source separation d, the length scale and intensity of the turbulence, and the molecular diffusivities of the two scalars. As described by Yee and Wilson [15], σ_a, σ_r, and $\imath_r$, which already are required as inputs for the standard fluctuating plume model, can be specified in terms of

a set of coupled ordinary differential equations derived from second-moment closures that model their development with downstream distance, x, from the source.

We will focus briefly on the modelling of the remaining two parameters; namely, r and r^*. Because the variations in the instantaneous plume centroid positions arise from the bulk meander component, it is reasonable to expect the correlation coefficient r to depend on the ratio of the source separation d to the outer plume length scale σ_a. Thus, we approximate r as an exponential with the form

$$r = \exp\left(-\frac{c_1 d}{\sigma_a}\right), \tag{19}$$

where c_1 is a model constant. Note that in Equation (19), the dependence of r on x is implicitly embodied in the outer plume length scale σ_a. Because we are interested primarily in trying to capture qualitative features in experimental data using our model, we will set order-unity numerical model constants such as c_1 identically equal to unity here in order to minimize the number of empirical constants (and, other configuration-dependent inputs) required to define the model.

The correlation coefficient, r^*, arising from the turbulent mixing of two passive scalars with either equal or unequal diffusivities, depends on a complicated physical process involving the stretching and folding of contaminant elements by the turbulence. When a blob of tracer is injected into a turbulent flow, it quickly deforms into thin folded strands and sheets. Turbulent fluid motions which repeatedly stretch and fold the contaminant elements serve also to increase the contact area for interdiffusion and, hence, to enhance the molecular mixing, and to homogenize the plume concentration as the large spatial gradients that are generated by the turbulent stirring are rapidly smeared out by molecular diffusion. This mixing process leads eventually to a stronger cross-correlation, r^*, between the two overlapping scalar concentrations (assuming equal molecular diffusivities for the two scalars). A number of approaches have been proposed to model the mixing of two scalars (and their concomitant effects in terms of the correlation coefficient r^*). These include the time-evolving mapping closure methodology (Gao and O'Brien [19]; Girimaji [20]) and conditional moment closure (CMC) theory (Bilger [21]). CMC theory has also been applied to the problem of the modelling of differential diffusion which involves unequal mixing between two scalars due to different molecular diffusivities (Kronenburg and Bilger [22]). Despite these efforts, it appears that our present understanding of molecular mixing

between scalars (of either equal or unequal diffusivities) is still very limited and the use of these models to predict r^* is currently only partially satisfactory at best.

III. HIGHER-ORDER CORRELATION FUNCTIONS

If a passive scalar $\phi(\mathbf{x}, t)$ is advected by a turbulent velocity field $\mathbf{v}(\mathbf{x}, t)$, it satisfies the advection-diffusion equation

$$\frac{\partial \phi}{\partial t} + (\mathbf{v} \cdot \nabla)\phi = D\nabla^2\phi, \tag{20}$$

where D is the coefficient of molecular diffusivity. The linearity of the advection-diffusion equation implies that the instantaneous concentrations from two separated sources are linearly superposable as are the mean concentrations (viz., $\phi_T = \phi_1 + \phi_2$, where ϕ_T is the total instantaneous concentration; also, $\langle \phi_T \rangle = \langle \phi_1 \rangle + \langle \phi_2 \rangle$). However, it is apparent that higher-order concentration moments are not linearly superposable. Indeed, in order to determine the higher moments of the total instantaneous concentration from the concentration moments of the individual instantaneous concentrations, it is necessary to know also joint statistics between the individual concentrations in the form of the higher-order correlation functions. In other words, a complete statistical description of the total instantaneous concentration resulting from two sources cannot be obtained separately from the information about the individual instantaneous concentrations because the additional information required about their joint behavior is not available.

For example, the central second-order moment (or, variance) of the total instantaneous concentration is given in terms of the variances of the individual instantaneous concentrations as follows:

$$\frac{\langle \phi_T'^2 \rangle - \langle \phi_1'^2 \rangle - \langle \phi_2'^2 \rangle}{2\langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2}} = \rho^{[1|1]}, \tag{21a}$$

where primed quantities are used to indicate a departure from the corresponding mean value. Equation (21a) implies that in order to determine the total concentration variance, we need information on the individual concentration variances as well as the second-order correlation function $\rho^{[1|1]} \equiv \rho$ defined as

$$\rho^{[1|1]} \equiv \rho \equiv \frac{\langle \phi_1' \phi_2' \rangle}{\langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2}} = \frac{\langle \phi_1 \phi_2 \rangle - \langle \phi_1 \rangle \langle \phi_2 \rangle}{\langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2}}. \tag{21b}$$

A perusal of Equation (21b) indicates that $\rho^{[1|1]}$ can be calculated using Equations (16), (17) and (18) with $l = m = 1$ to determine the joint concentration moment $\langle \phi_1 \phi_2 \rangle$, and Equation (6) with $n = 1$ and 2 to determine the first and second concentration moments for the individual sources taking into account that $z \rightarrow z + d/2$ for source 1 and $z \rightarrow z - d/2$ for source 2 in the application of Equation (6) (i.e., recall that the origin of the Cartesian coordinate system for two sources separated by a distance d lies midway between the sources, whereas the origin of the Cartesian coordinate system for a single source coincides with the location of the source).

It is straightforward to extend these ideas to higher concentration moments. To this purpose, it is convenient to define a mixed $(l+m)$th order correlation function as

$$\rho^{(l,m)} \equiv \frac{\langle \phi_1'^l \phi_2'^m \rangle}{\langle \phi_1'^{2l} \rangle^{1/2} \langle \phi_2'^{2m} \rangle^{1/2}}. \tag{22}$$

With this definition, it is noted that $\rho^{(1,1)} = \rho^{[1|1]} \equiv \rho$ is simply the second-order correlation function, which is the key joint concentration statistic required to determine the total concentration variance given the individual concentration variances [cf. Equation (21)]. Similarly, it is easy to verify that the central third-order moment of the total instantaneous concentration is related to the moments of the individual instantaneous concentrations as follows:

$$\frac{\langle \phi_T'^3 \rangle - \langle \phi_1'^3 \rangle - \langle \phi_2'^3 \rangle}{3\left(\langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2} + \langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2} \right)} = \rho^{[1|2]}, \tag{23a}$$

where $\rho^{[1|2]}$ is a third-order correlation function defined as the following linear combination of the mixed $(1,2)$ and $(2,1)$ correlation functions:

$$\rho^{[1|2]} \equiv f_0 \rho^{(1,2)} + (1 - f_0) \rho^{(2,1)}, \tag{23b}$$

with

$$f_0 \equiv \frac{\langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2}}{\langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2} + \langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2}}. \tag{23c}$$

Furthermore, the central fourth-order moment of the total instantaneous concentration can be determined from the moments of the individual instantaneous concentrations using the relationship

$$\frac{\langle \phi_T'^4 \rangle - \langle \phi_1'^4 \rangle - \langle \phi_2'^4 \rangle}{10\left(\langle \phi_1'^6 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2} + \langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2} + \langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^6 \rangle^{1/2} \right)} = \rho^{[1|2|3]}, \tag{24a}$$

where $\rho^{[1|2|3]}$ is the fourth-order correlation function defined as a particular linear combination of the mixed $(1,3)$, $(3,1)$, and $(2,2)$ correlation functions:

$$\rho^{[1|2|3]} = \frac{2}{5} f_1 \rho^{(3,1)} + \frac{3}{5} f_2 \rho^{(2,2)} + \frac{2}{5}(1 - f_1 - f_2)\rho^{(1,3)}, \tag{24b}$$

with

$$f_1 \equiv \frac{\langle \phi_1'^6 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2}}{\left(\langle \phi_1'^6 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2} + \langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2} + \langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^6 \rangle^{1/2} \right)}, \tag{24c}$$

and

$$f_2 \equiv \frac{\langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2}}{\left(\langle \phi_1'^6 \rangle^{1/2} \langle \phi_2'^2 \rangle^{1/2} + \langle \phi_1'^4 \rangle^{1/2} \langle \phi_2'^4 \rangle^{1/2} + \langle \phi_1'^2 \rangle^{1/2} \langle \phi_2'^6 \rangle^{1/2} \right)}. \tag{24d}$$

It should be pointed out that $\rho^{[1|2]}$ and $\rho^{[1|2|3]}$ play the same role with respect to the determination of the central third and fourth moments of the total instantaneous concentration from the moments of the individual instantaneous concentrations as does the second-order correlation $\rho \equiv \rho^{[1|1]}$ with respect to the concentration variance. Positive (negative) values for these higher-order correlation functions imply a constructive (destructive) interference in the associated higher-order central concentration moment field in the sense that the total higher-order concentration moment is enhanced (reduced) relative to the corresponding concentration moment obtain by simple addition (linear superposition) of the individual concentration moments. Furthermore, all the higher-order correlation functions defined here can be determined by applying Equations (16), (17), (18) and (6) for the appropriate choice of values for l, m, and n and by using the standard relations between the moments about the origin and the central moments of random functions.

Figures 1 to 4 present the shapes of the transverse profiles of the second-order correlation ρ in the z-direction (viz., in the direction of the source separation) for different spacings, d, between the sources and for various combinations of the plume meander ratio, M, fluctuation intensity in the relative frame, i_r, and concentration correlation coefficient for in-plume mixing, r^*. For all the model results, r is assumed to be given by Equation (19) with $c_1 \equiv 1$. As to be expected, $\rho \to 0$ as either $d \to \infty$ (i.e., the two sources are acting independently of each other) or $z \to \infty$ at a fixed value of d. If the bulk meandering is relatively weak ($M \lesssim 1$) and/or internal fluctuation intensity is small ($i_r \lesssim 1$), then ρ generally exhibits a distinct minimum along the centerline between the two plumes, but this minimum tends to become less pronounced as the source separation, d, decreases and/or

as M or i_r increases. These general trends agree qualitatively with the measurements of Warhaft [11]. Indeed, for $M \ll 1$ and $i_r \ll 1$, the correlation along the centerline between the two plumes can become significantly negative [cf. Figure 4(d)] despite a large positive value for r^* (i.e., a strong in-plume scalar mixing implying $r^* \approx 1$) and increase strongly in correlation towards a maximum near the centerline of either plume before decreasing (albeit slowly) towards zero correlation as $z \rightarrow \pm\infty$. For strong meandering and/or strong in-plume concentration fluctuation levels, the maximum correlation can occur along the centerline between the two plumes with a decrease towards zero in the correlation as one moves away from this centerline location.

Model transverse profiles of the third-order correlation $\rho^{[1|2]}$ are exhibited in Figures 5 to 8 for a range of values of M, i_r, r_*, and d. Generally, for plumes exhibiting strong meander, the shapes of the transverse profiles of $\rho^{[1|2]}$ are similar to those of ρ. However, for larger instantaneous plumes with weaker meandering motions ($M \ll 1$), the variation of $\rho^{[1|2]}$ with transverse position is considerably more complicated than that exhibited by ρ. In particular, for both weak meandering and small in-plume fluctuations [cf. Figures 7(d) and 8(d)], the variation of $\rho^{[1|2]}$ with transverse position is extremely complicated: for a certain range of spacings between the sources relative to the mean instantaneous width of each plume ($d/\sigma_r \lesssim 1$), $\rho^{[1|2]}$ has a relative maximum at the centerline between the two plumes, decreases to a relative minimum near the centerline of either plume, increases again to a maximum near the fringes of either plume, before decreasing towards zero as $z \rightarrow \pm\infty$.

Transverse profiles for the fourth-order correlation $\rho^{[1|2|3]}$ are shown in Figures 9 to 12 for various values of M, i_r, r^*, and d. Again, the shapes of these profiles are similar to those of $\rho^{[1|2]}$ and ρ for strongly meandering plumes. Interestingly, a comparison of Figures 5 to 8 with Figures 9 to 12 shows that there are plume states (as characterized by M and i_r) for which the total central third concentration moment field at certain receptor positions exhibits destructive interference, whereas the total central fourth concentration moment field at these same positions exhibits constructive interference [e.g., cf. Figure 7(d) with Figure 11(d)].

The dependence of ρ, $\rho^{[1|2]}$, and $\rho^{[1|2|3]}$ at the centerline between the two sources on the source separation, d, is shown in Figure 13 for different values of the meander ratio, M, with $i_r = 0.05$ (weak internal fluctuations) and $r^* = 0.95$ (strong in-plume mixing between the two plume concentrations). It is seen that ρ is positive at small d where the

plumes are initially flapped in unison, then decreases and becomes negative at intermediate values of d, and finally increases again at large values of d towards zero as the two plumes become independent of one another. These principal qualitative features in ρ have been observed in the measurements of Warhaft [11] in grid turbulence and of Sawford et al. [12] in atmospheric surface-layer turbulence. Furthermore, the maximum value of the negative correlation in ρ and the source separation d at which this value occurs both increase as M decreases. Interestingly, for weaker meandering ($M \lesssim 1$) and small d, ρ and $\rho^{[1|2|3]}$ are both positive implying a constructive interference in the concentration variance and central fourth concentration moment fields, whereas $\rho^{[1|2]}$ is negative implying a destructive interference in the central third concentration moment field. This behavior is exactly reversed for a range of intermediate source separations for the case of the interference of large instantaneous plumes that meander little ($M \ll 1$). However, for stronger meandering motions in the plume (e.g , $M \gtrsim 1/25$), $\rho^{[1|2]}$ and $\rho^{[1|2|3]}$ are positive for the range of intermediate values of d, whereas ρ is negative in this range. Hence, a constructive (destructive) interference in the concentration variance field does not necessarily imply a constructive (destructive) interference in all the higher-order concentration moment fields.

IV. COMPARISON WITH WATER CHANNEL MEASUREMENTS

Here we make comparisons between the theoretical and measured correlation functions of the partially overlapping concentration fields generated by two separated point sources in grid turbulence. The emphasis here is to assess the ability of the model to capture major trends and qualitative features in the experimental data, rather than precise quantitative agreement. In consequence, we are not concerned with the fine tuning of model parameters in order to optimize the fit with the data.

IV.1. Experimental details

Yee and Wilson [15] describe measurements of the plume concentration field downstream of an isolated point source in decaying homogeneous turbulence (grid turbulence in a water channel). The present experimental method involving two separated point sources in grid turbulence is similar to that used for the isolated point source measurements.

The measurements were conducted in a recirculating water channel with a rectan-

gular cross-section 0.420 m high (at the water surface), 0.680 m wide and 5 m long with an active test section of 3 m. The turbulence was generated by a biplanar arrangement of rectangular-sectioned flat bars 19.2 mm wide and 5.0 mm thick with their centers spaced $L_G = 76.2$ mm apart (i.e., mesh length) to give a grid solidity of 0.44. The mean channel (free-stream) speed U was 188 mm s^{-1}. At this average channel speed, the grid Reynolds number $Re \equiv UL_G/\nu$ (where ν is the kinematic viscosity of water) is 14300.

The measured flow statistics (e.g., mean velocity U and streamwise turbulence intensity σ_u/U) were approximately constant in a transverse plane through the water channel, implying that the flow field is that of normal grid flow. The decay of the streamwise and vertical velocity variances, σ_u^2 and σ_w^2, respectively, were found to be well approximated by the following power laws:

$$\frac{\sigma_u^2}{U^2} = 0.0209\left(\frac{x}{L_G} + 6.36\right)^{-1}, \tag{25a}$$

and

$$\frac{\sigma_w^2}{U^2} = 0.0169\left(\frac{x}{L_G} + 6.36\right)^{-1}. \tag{25b}$$

The cross-stream turbulence velocity variance σ_v^2 was not measured. Transverse isotropy with $\sigma_w/\sigma_v = 1$ was assumed, and confirmed by the equality of measured plume spreads in the cross-stream (y) and vertical (z) directions. The streamwise flow statistics are slightly anisotropic with respect to those in the transverse plane with $\sigma_v/\sigma_u \equiv \sigma_w/\sigma_u \approx 0.90$.

Within the range of streamwise measurements ($x = 500$ to 2000 mm), the average value of the integral length scale, Λ, of turbulence was found to be 27.5 mm. The integral scale was computed directly from the velocity fluctuation data by determining the smoothing effect of increasing time averaging on the velocity time series (Wilson et al. [23]) and invoking Taylor's frozen turbulence hypothesis.

In the experiments, the tracer consisted of a neutrally buoyant mixture of salt water and ethanol (50 g l^{-1} NaCl), the former being used as the tracer for the concentration measurements. It should be pointed out that salt is a weakly diffusive scalar in water with a Schmidt Number $Sc \equiv \nu/D$ (D is the molecular diffusivity of salt in water) of about 830. In the dual source experiments, salt tracer was injected into the flow from a pair of coflowing point sources (38.1 mm long horizontal tubes, each with a 4.19 mm O.D. and 3.45 mm I.D., aligned with the flow) for two different source separations ($d = 40$ and 100 mm) in the vertical (z) direction. The experimental procedure involved operating each source

separately and then both together, with measurements made of the vertical plume cross-sections of the instantaneous saline concentrations at four different downstream locations from the sources (x = 500, 1000, 1500, and 2000 mm; or, x/Λ = 18.2, 36.4, 54.5, and 72.7). Fluctuating concentration measurements in the saline plume were obtained with a rake of 8 fast-response electrical conductivity probes with a 20 mm separation between probe tips.

IV.2. Results

Here we present measured correlation functions between the saline concentrations of the two partially overlapping plumes and compare them with the model predictions. As noted earlier, the model requires as input 5 parameters: namely, σ_a, σ_r, i_r (or, equivalently k), r, and r^*. For this study, the first three parameters were calculated from the measured grid turbulence data using the following system of transport equations for the outer and inner plume length scales and for the fluctuation intensity in the relative frame (Yee and Wilson [15]):

$$\frac{d\sigma_a^2}{dx} = 2K_a'; \tag{26a}$$

$$\frac{dK_a'}{dx} = \frac{\sigma_w^2}{U^2} - \frac{A}{\Lambda}\left(\frac{q}{U}\right)K_a'; \tag{26b}$$

$$\frac{d\sigma_r^2}{dx} = 2K_r'; \tag{26c}$$

$$\frac{dK_r'}{dx} = \frac{\sigma_w^2}{U^2}(l_w) - \frac{A}{l_w}\frac{q(l_w)}{U}K_r'; \tag{26d}$$

$$\frac{\sigma_w^2}{U^2}(l_w) \equiv \frac{\sigma_w^2}{U^2}\left(\frac{\min(l_w,\Lambda)}{\Lambda}\right)^{2/3}; \tag{26e}$$

$$\frac{q(l_w)}{U} \equiv \frac{q}{U}\left(\frac{\min(l_w,\Lambda)}{\Lambda}\right)^{1/3}; \tag{26f}$$

$$l_w \equiv \min(\alpha_1\sigma_r, \Lambda); \tag{26g}$$

$$\frac{d}{dx}\left(\frac{i_r^2}{\sigma_r^2}\right) + \frac{\alpha_c}{\Lambda_c}\left(\frac{q_c}{U}\right)\left(\frac{i_r^2}{\sigma_r^2}\right) = -\frac{d}{dx}\left(\frac{1}{\sigma_r^2}\right); \tag{26h}$$

$$q_c = q\left(\frac{\Lambda_c}{\Lambda}\right)^{1/3} H\big(-(\Lambda_c - c_0\Lambda)\big) + qH(\Lambda_c - c_0\Lambda); \tag{26i}$$

$$\frac{d\Lambda_c}{dx} = a_1\frac{q_c}{U}H\big(-(\Lambda_c - c_0\Lambda)\big) + b_1\left(\frac{q}{U}\right)\left(\frac{\Lambda}{\Lambda_c}\right)H(\Lambda_c - c_0\Lambda); \tag{26j}$$

and

$$\alpha_c = a_2 H\big(-(\Lambda_c - c_0\Lambda)\big) + b_2 H\big(\Lambda_c - c_0\Lambda\big). \qquad (26k)$$

Here, $H(x)$ is the unit step function: $H(x)$ is zero for $x < 0$ and unity for $x > 0$; $q \equiv (\sigma_u^2 + \sigma_v^2 + \sigma_w^2)^{1/2}$; and, q_c and Λ_c are the velocity and length scales of concentration fluctuations. Equations (26i), (26j), and (26k) model the velocity and length scales of concentration fluctuations required in a fluctuation-dissipation relation that relates the scalar dissipation ϵ_c to the concentration variance $\langle \phi'^2 \rangle$. This model was originally proposed by Sykes et al. [24], but the model constants used here are different. In Equation (26), the model constants are: $A = 0.75$, $\alpha_1 = 1.0$, $a_1 = 0.30$, $a_2 = 0.60$, $c_0 = 0.66$, $b_1 = 0.17$, and $b_2 = 0.52$. These constants have been chosen to produce approximately the correct spreading rate and centerline fluctuation intensity in relative coordinates for the isolated source releases (Yee and Wilson [15]).

Once σ_a has been determined, r can be calculated using Equation (19) for a given source separation d (assuming $c_1 = 1$ since we are interested principally in qualitative features of the model). No transport model has been used here to determine r^* as a function of downstream distance, x. Rather, for comparisons with our limited experimental data, we shall treat r^* simply as a free numerical constant whose value will be fixed to optimize quantitative agreement with the data. The justification for treating the parameter r^* as a constant here, rather than as a function of x, will be provided below.

Figure 14(a) shows the downstream development of σ_a, σ_r, and $\sigma_m \equiv (\sigma_a^2 - \sigma_r^2)^{1/2}$. Figure 14(b) presents i_r and $M \equiv \sigma_m^2/\sigma_r^2$ as a function of distance from the source. The downstream evolution of these parameters were obtained by numerically integrating the system of ordinary differential equations summarized in Equation (26) for a source of initial size $\sigma_s/\Lambda = 0.0436$ ($\sigma_s = 1.2$ mm), which was estimated using a mass balance method as described in Yee and Wilson [15]. A perusal of Figure 14 shows that this system of transport equations gives values for $\sigma_a(x)$, $\sigma_r(x)$, and $i_r(x)$ that agree very well with the data. In particular, these parameters are calculated to within experimental error. An interesting feature in the data, which is reflected also in the model predictions, is the monotonic increase in i_r over the entire downstream range of our measurements [cf. Figure 14(b)]. This increase in the fluctuation intensity suggests that mixing of the saline tracer with the ambient fluid (water) occurs very slowly. This is simply a consequence of the low mass diffusivity of the saline tracer and of the release of the tracer into grid-generated turbulence (i.e., a flow which

is homogeneous and without shear), both effects which are conducive to an attenuated rate of mixing Hence, in our case, we would expect r^* (which is a descriptor of the in-plume mixing processes) to evolve only very slowly with downstream distance. Consequently, over the limited streamwise range corresponding to our measurements, it would seem reasonable to treat r^* simply as a constant parameter. Furthermore, the putative slow evolution of mixing between scalars suggests that asymptotic values of parameters such as s_r and r^* would be difficult to determine in grid turbulence in a water channel given its necessarily limited downstream fetch.

Figure 15 shows comparisons between measurements and model predictions for the second-order correlation function ρ between the saline plume concentrations at four downstream distances x for two source separations d. The measured values for ρ were obtained from the inference method (Warhaft [11]) in which the correlation was determined from independent measurements of the concentration variance profiles for each source operating separately and for both sources operating together. These three concentration variance profiles were then substituted into Equation (21a) to provide an estimate for ρ. It is important to point out that statistical error is inevitable in the determination of ρ by the inference method, especially when $|\rho| \ll 1$. In particular, small errors in the estimation of $\langle \phi_T'^2 \rangle$, $\langle \phi_1'^2 \rangle$, and $\langle \phi_2'^2 \rangle$ can lead to large errors in the estimation of ρ especially when $|\rho| \ll 1$ because ρ in this case depends on the difference between two nearly equal quantities (i.e., $\langle \phi_T'^2 \rangle$ and $\langle \phi_1'^2 \rangle + \langle \phi_2'^2 \rangle$). Hence, the accurate determination of ρ by the inference method becomes rather difficult for large source separations d. In principle, the higher-order correlations such as $\rho^{[1|2]}$ and $\rho^{[1|2|3]}$ can be determined by using Equations (23a) and (24a) in an natural extension of Warhaft's original proposed inference method. However, for our present data, the higher-order concentration moments (up to sixth-order) were not determined sufficiently accurately to enable the reliable determination of the third- and fourth-order correlation functions using this generalized inference method.

For the model predictions, r^* was taken to be 0.5. This value for r^* was found to give the best agreement with our measurements of ρ over the available downstream range covered by the data. No reliable measurements of ρ were obtained for the source separation d of 100 mm at $x/\Lambda = 18.2$. Generally, the measurements of ρ for both $d = 40$ and 100 mm at the four downstream locations are in reasonable agreement with the model predictions for $r^* = 0.5$, given the inevitable experimental scatter to be expected in these difficult measurements. However, it should be stressed that considering the difficulties in

making accurate measurements of ρ using the inference method (especially when d is large and/or when the receptor position is near the edges of the plumes) and considering the scatter in the measurements, it is not possible to draw firm conclusions about the absolute (quantitative) accuracy of the model calculation of ρ. The present measurements for ρ show a maximum in the correlation along the centerline between our two saline plumes, whereas the measurements by Warhaft [11] show a minimum in the correlation along the centerline between his thermal plumes. This disparity in behavior can be attributed to differences in both the degree of bulk meandering and the level of internal concentration fluctuations between the two situations. The degree of bulk meandering is larger in our saline plumes than in Warhaft's thermal plumes because the saline plumes were generated from point sources, whereas the thermal plumes were generated from line sources. Furthermore, the internal concentration fluctuation level is expected to be much larger in the saline plumes than in the thermal plumes because molecular diffusion effects of salt in water ($Sc \approx 830$) are much smaller than those of heat in air ($Sc \approx 0.7$).

Figure 16 compares measured centerline [$(y, z) = (0, 0)$] streamwise profiles of ρ with the model predictions for the two source separations d. It is seen that the model predictions show very similar behavior to the measurements. Furthermore, we note that there are qualitative differences between the streamwise evolution of ρ observed in our saline plumes and those observed by Warhaft [11] in his thermal plumes. Although the monotonic increase in ρ with downstream distance x observed in our saline plumes is qualitatively similar to that observed in Warhaft's thermal plumes for smaller d ($d \lesssim 20$ mm), the change in the evolutionary behavior of ρ with x at larger d (i.e., ρ decreases to a minimum with increasing x and increases again at larger x), evident in Warhaft's thermal plumes, is not seen in our saline plumes. Again, it is likely that these discrepancies in behavior can be attributed to the differences in bulk meandering and internal concentration fluctuation level between the two situations.

V. CONCLUSIONS

The fluctuating plume model (i.e., a meandering plume model incorporating internal fluctuations) which models the statistical characteristics of the plume concentration field generated downstream of an isolated point source release has been generalized to model the interference in the concentration fields produced by a pair of separated point sources.

In this natural generalization, a fundamental relation based on a statistical law has been used to relate the joint concentration PDF for the two sources in the absolute frame to the convolution of the joint spatial PDF of the positions of the centroids of the two plumes and the joint concentration PDF in the relative frame. In the application of this fundamental relation, the joint spatial PDF of the plume centroid positions has been assumed to be a multivariate Gaussian distribution, and the joint concentration PDF in relative coordinates has been assumed to be reasonably approximated by a bivariate gamma distribution. Using the fundamental relation with these assumptions, closed-form expressions for the joint concentration moments of arbitrary integral orders between the two scalars have been derived explicitly. This result, when combined with the analytical expressions for the higher-order concentration moments obtained from the standard fluctuating plume model of Yee and Wilson [15], has been used to obtain analytical results for the higher-order correlation functions between the concentration fluctuations produced by the two-point-source configuration.

The results for the higher-order correlation functions require as input 5 parameters that are inherited from the generalized fluctuating plume model: (1) the absolute plume dispersion, σ_a, which determines the outer length scale for plume growth; (2) the relative plume dispersion, σ_r, which determines the inner length scale for plume growth; (3) the fluctuation intensity, i_r, in the relative coordinates which measures the importance of the in-plume structure contribution to the concentration fluctuations; (4) the correlation coefficient, r, between the positions of the instantaneous centroids of the two plumes; and, (5) the correlation coefficient, r^*, between the concentration fluctuations of the two plumes in relative coordinates. It should be pointed out that each of these parameters has a good physical justification and can be related phenomenologically either to the physics of plume dispersion and/or to the interference of two plumes Explicit model results have been presented for transverse profiles of the second-, third-, and fourth-order correlation between the concentration fluctuations for a pair of point sources for a range of values of the physical parameters and the source separation, d. These model results encompass a surprisingly rich and complex structure, exhibiting a wide variation in form of the higher-order correlation functions. Over certain ranges of the physical parameters, the second-order correlation functions appear to be qualitatively similar to measurements obtained by Warhaft [11] in homogeneous turbulence in a wind tunnel and by Sawford et al. [12] in inhomogeneous turbulence in the atmospheric surface layer.

The model for the second-order correlation function has been compared to some new measurements obtained for a pair of separated sources in grid turbulence in a water channel. Although the principal objective here was to use the model to obtain a qualitative understanding of the water channel measurements rather than to achieve a more demanding quantitative prediction, it has been shown, nevertheless, that the model produces a reasonable agreement with the experimental results provided the correlation coefficient, r^*, which reflects the state of in-plume mixing between the overlapping concentrations of the two plumes, is chosen to be about 0.5. Interestingly, the shapes of the second-order correlation, ρ, in the transverse plane for the present water channel measurements of two interfering saline plumes is different than that obtained by Warhaft [11] for two interfering thermal plumes in a wind tunnel. In particular, ρ for the thermal plumes in the wind tunnel exhibits a minimum at a point midway between the two sources and increases towards either source. In contrast, ρ for the two interfering saline plumes in the water channel exhibits a maximum at a point midway between the sources and decreases towards either source. The differences can be attributed to the degree of bulk meandering and the level of internal plume fluctuations between the two cases. The former effect is the result of the geometry of the sources (i.e., point sources in the water channel versus line sources in the wind tunnel) and the latter effect is the result of the differences in the molecular diffusivities of the two tracers used (i.e, salt in water with $Sc \approx 830$ versus heat in air with $Sc \approx 0.7$). In either case, the model appears to predict the correct shapes for ρ (at least qualitatively, if not quantitatively).

Although r^* was found to be approximately constant over the range of streamwise measurements made in our water channel, it seems reasonable to expect that r^* in general would be dependent on x. This results from the fact that r^* is a measure of the degree of mixing between two scalars when they come into contact, and this degree of mixing must evolve with downstream distance. Indeed, for two scalars with equal molecular diffusivities, it is expected that with continued mixing with increasing x, the fluctuation intensity i_r will decay to zero and r^* will increase to unity in the far field ($M \rightarrow 0$) as the two scalars become completely mixed with each other (and, hence, perfectly correlated) in the final, fully-mixed asymptotic state characterizing the turbulent diffusive regime of plume development. The problem of mixing between two scalars becomes more complex, however, in the case of unequal molecular diffusivities. In this case, the phenomenon of differential diffusion can cause the two scalars to become decorrelated in the far field (i.e., $r^* \rightarrow 0$), although this

molecular effect is expected to weaken with increasing Reynolds number. In consequence, to represent the behavior of the higher-order correlation functions over the whole streamwise domain will require the specification of an evolution (transport) equation for the parameter r^*. It is expected that future work will concentrate on this aspect of the modelling.

Finally, measurements of the higher-order correlation between concentration fluctuations of two sources are presently not available. To verify (either qualitatively or quantitatively) the model features predicted to be present in the higher-order correlation will require some new measurements of passive scalar dispersion from two separated sources.

VI. LIST OF SYMBOLS

C_r	ensemble-mean concentration in relative coordinates
$C_{r,0}$	centerline C_r
$C_{r,j}$	C_r contributed by source j $(j = 1, 2)$
d	source separation
D	molecular diffusivity
f	concentration probability density function in absolute coordinates
f_c	plume centroid position probability density function
f_r	concentration probability density function in relative coordinates
${}_2F_1(\cdot)$	hypergeometric function
$H(\cdot)$	unit step function
i_r	concentration fluctuation intensity in relative coordinates
$I_n(\cdot)$	modified Bessel function of the first kind of order n
k	relative in-plume fluctuation measure: $k \equiv i_r^{-2}$
r	correlation coefficient of locations of plume centroids
r^*	correlation coefficient between concentrations in relative coordinates
L_G	biplanar grid mesh length
M	meander ratio: $M \equiv \sigma_m^2/\sigma_r^2$
q^2	twice the turbulence kinetic energy: $q^2 \equiv \sigma_u^2 + \sigma_v^2 + \sigma_w^2$
q_c	concentration fluctuation velocity scale
U	mean channel (free-stream) speed
x	downstream distance from source
y	cross-stream distance from source
y_c	cross-stream position of plume centroid

$y_c^{(j)}$	cross-stream position of plume centroid for source j ($j = 1, 2$)
z	vertical distance from midway between the sources
z_c	vertical position of plume centroid
$z_c^{(j)}$	vertical position of plume centroid for source j ($j = 1, 2$)
$\Gamma(\cdot)$	gamma function
Λ	integral length scale of turbulence
Λ_c	concentration fluctuation length scale
ν	kinematic viscosity
$\rho \equiv \rho^{[1\|1]}$	second-order correlation between concentration
$\rho^{[1\|2]}$	third-order correlation between concentration
$\rho^{[1\|2\|3]}$	fourth-order correlation between concentration
$\rho^{(l,m)}$	mixed $(l+m)$th order correlation function, Equation (22)
σ_a	absolute plume dispersion
σ_m	plume centroid dispersion
σ_r	relative plume dispersion
σ_u	standard deviation of streamwise velocity fluctuations
σ_v	standard deviation of cross-stream velocity fluctuations
σ_w	standard deviation of vertical velocity fluctuations
σ_s	initial source size
ϕ	instantaneous (random) concentration
ϕ_j	instantaneous concentration contributed by source j ($j = 1, 2$)
ϕ_T	instantaneous concentration contributed by both sources
ϕ'	concentration fluctuations
χ	sample concentration values that can be taken on by ϕ
χ_j	sample concentration values for source j ($j = 1, 2$)

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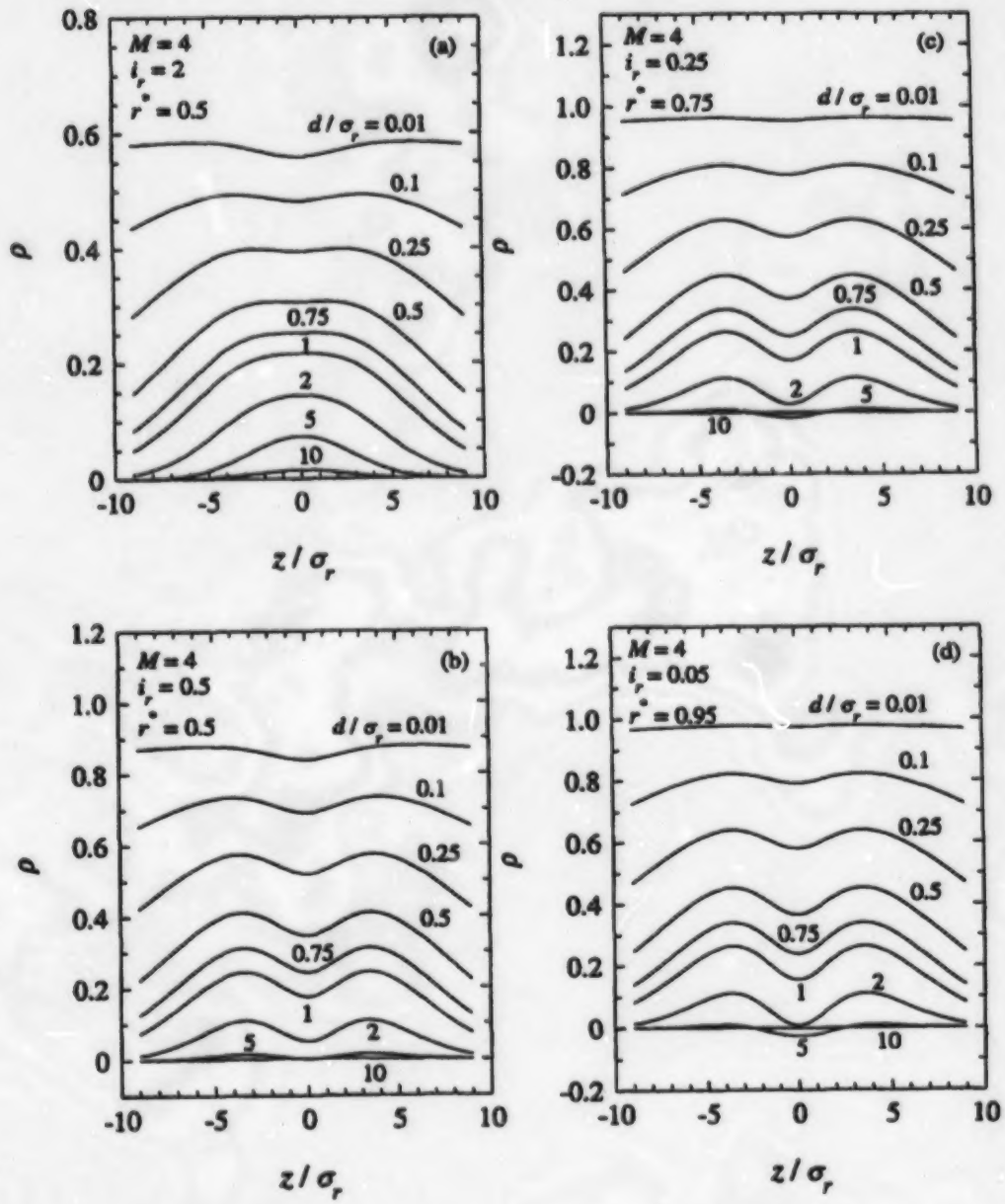

Figure 1. Transverse profiles of the second-order correlation coefficient, ρ, for $M = 4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

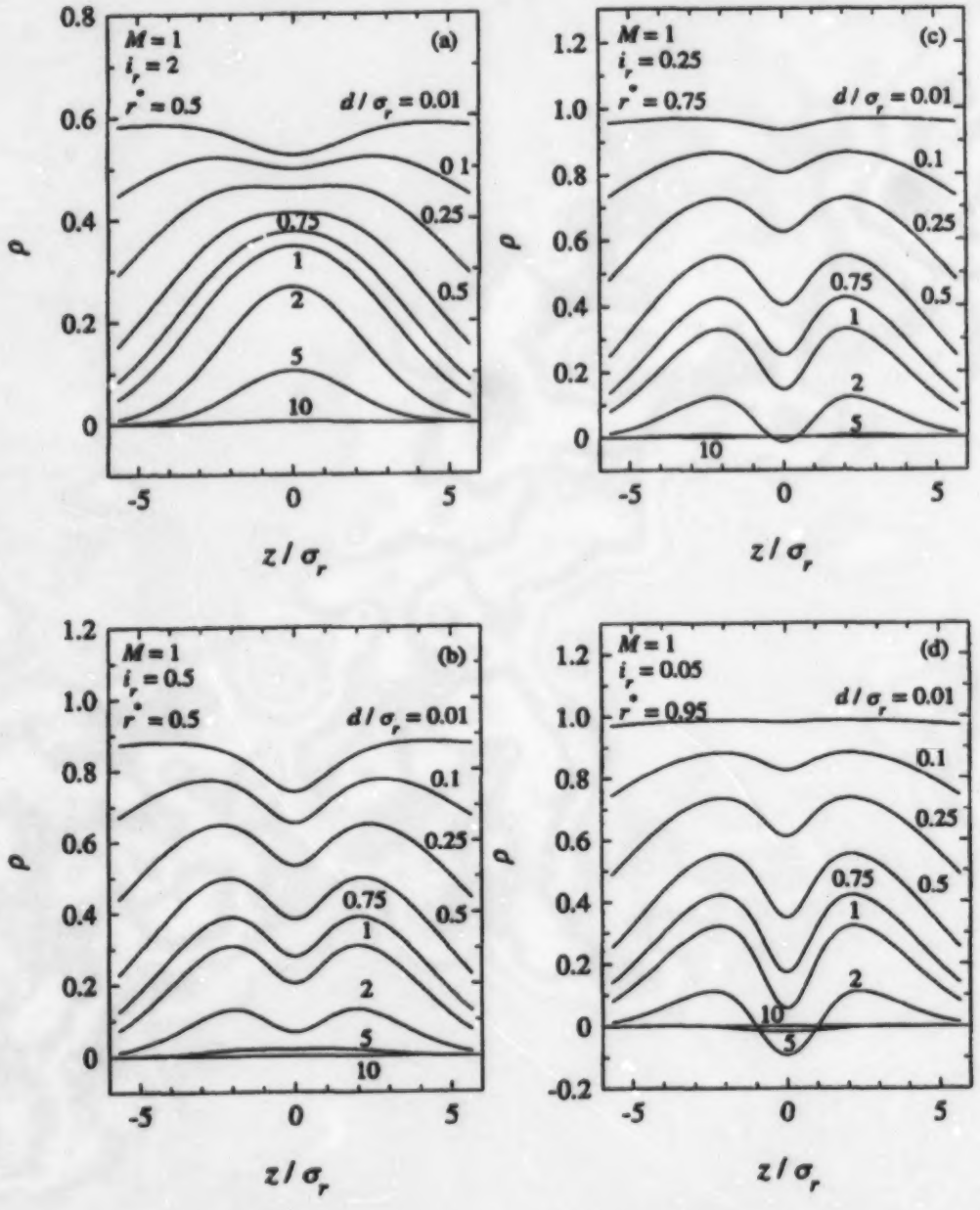

Figure 2. Transverse profiles of the second-order correlation coefficient, ρ, for $M = 1$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

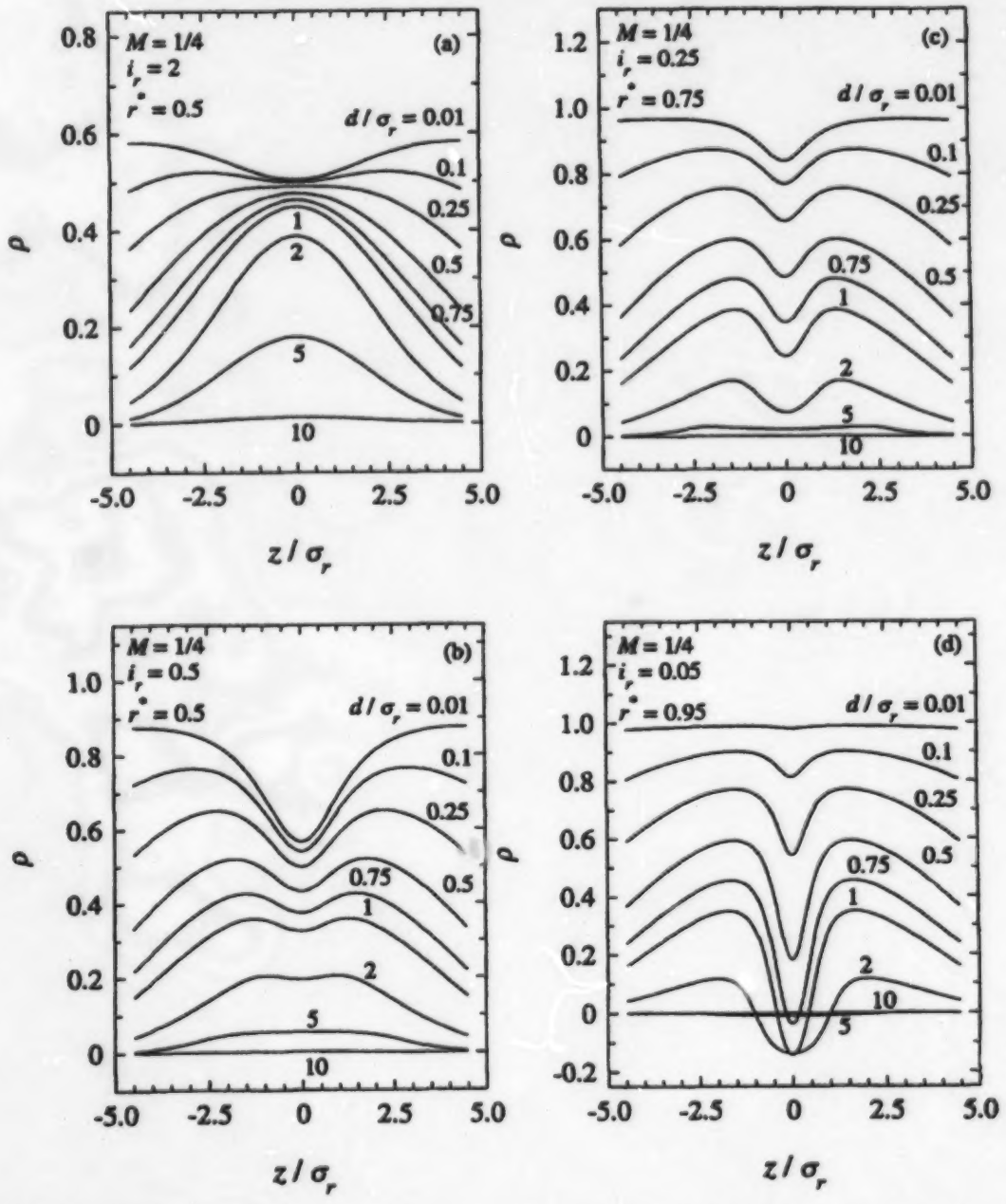

Figure 3. Transverse profiles of the second-order correlation coefficient, ρ, for $M = 1/4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

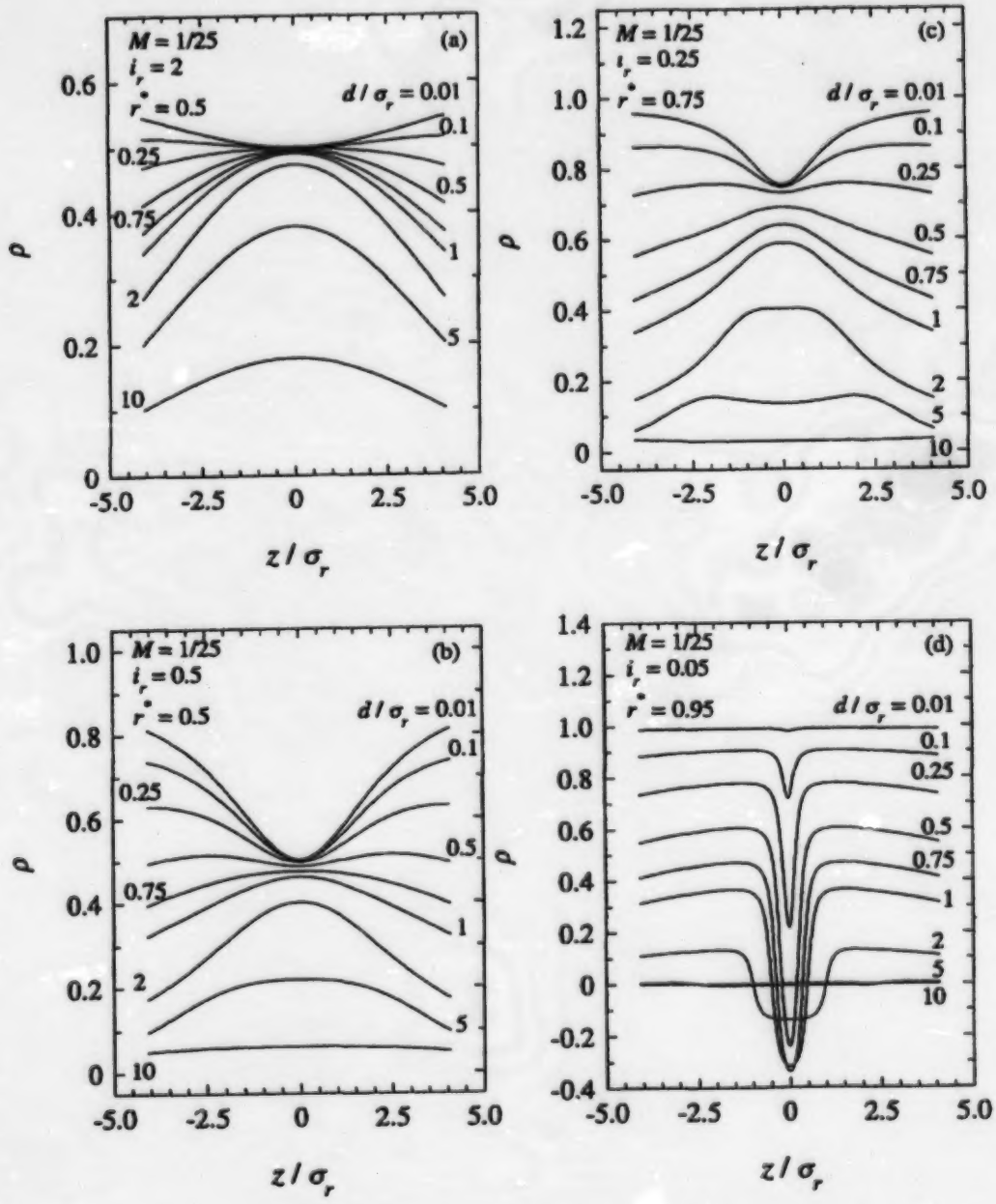

Figure 4. Transverse profiles of the second-order correlation coefficient, ρ, for $M = 1/25$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

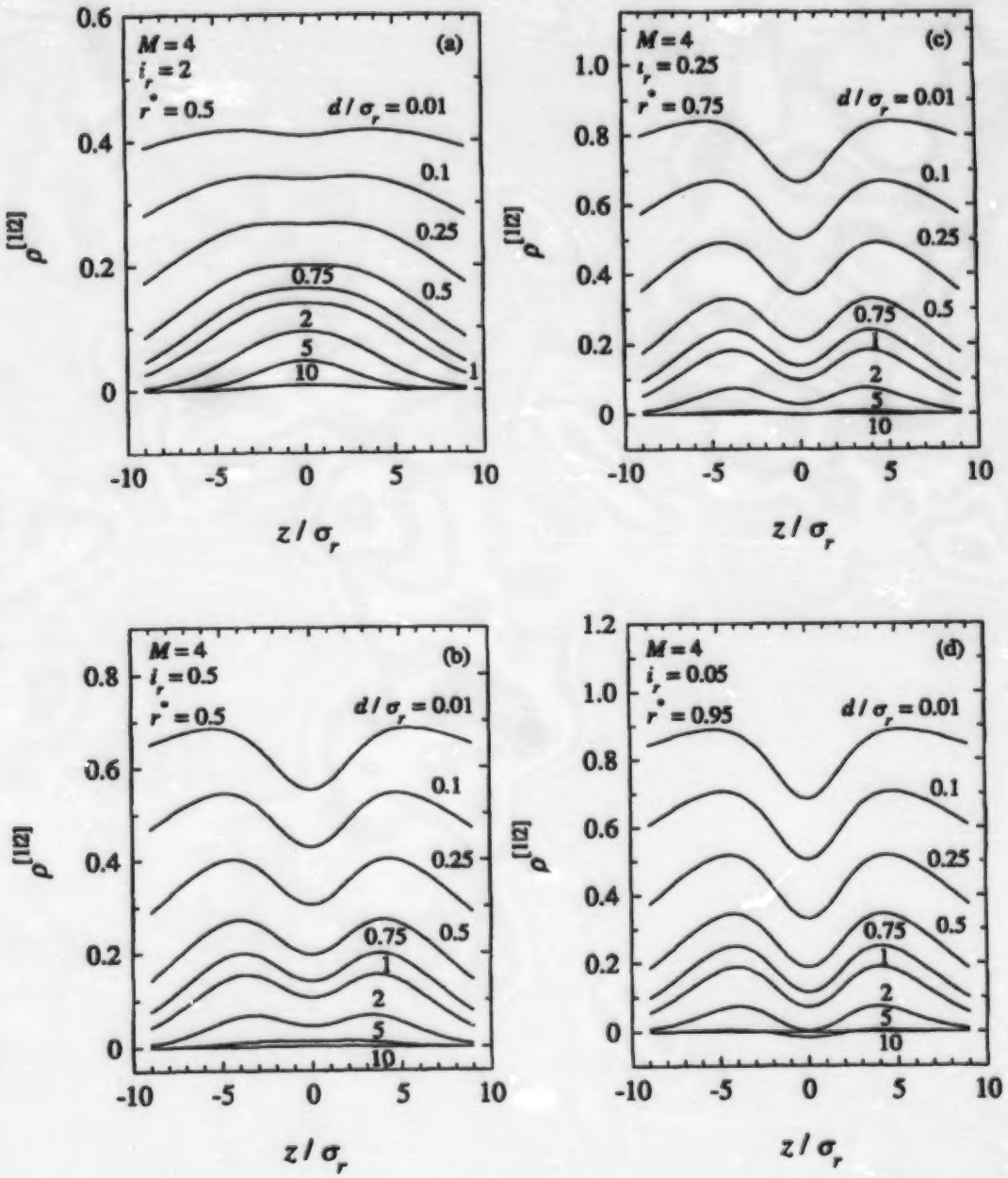

Figure 5. Transverse profiles of the third-order correlation coefficient, $\rho^{[12]}$, for $M = 4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

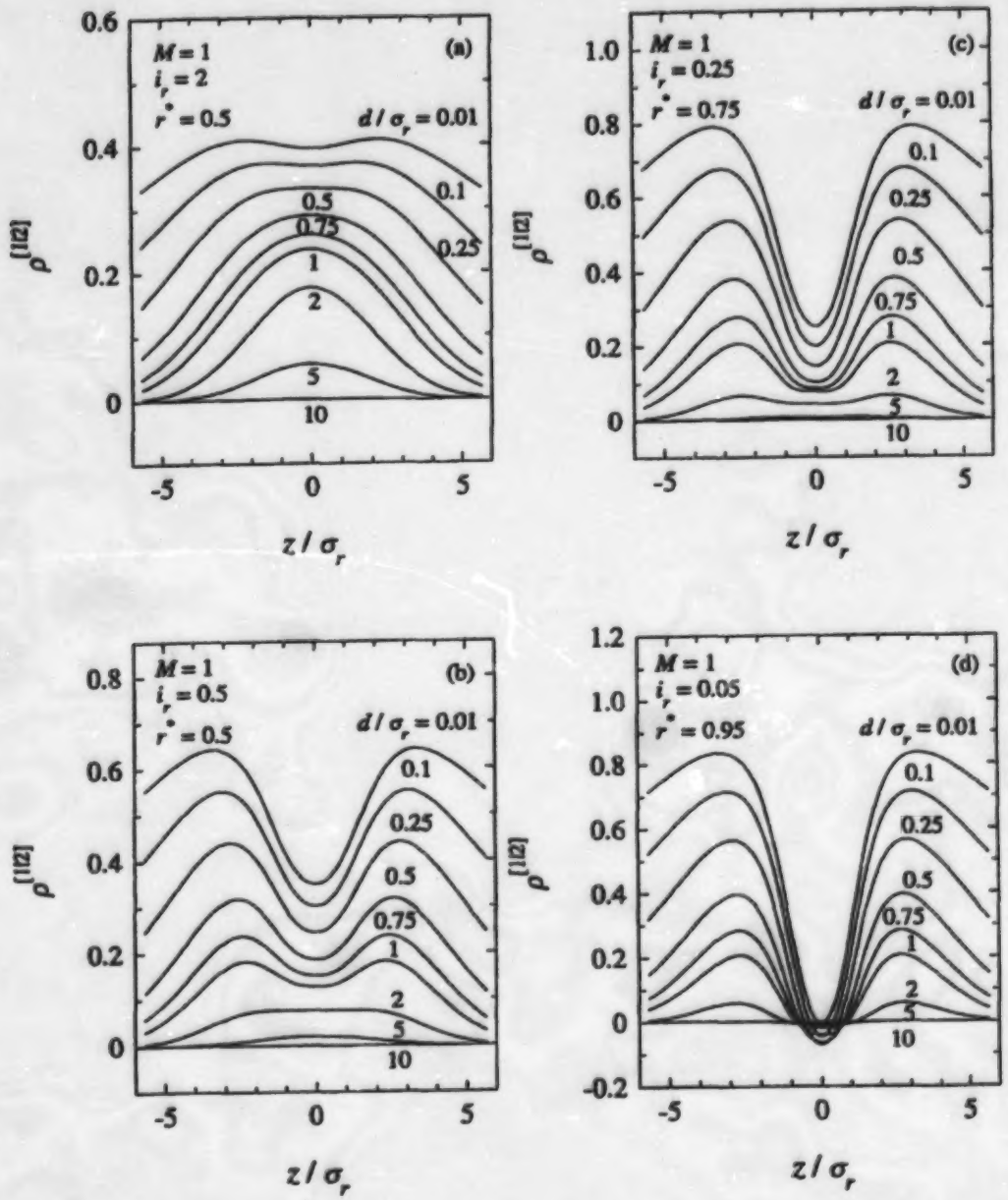

Figure 6. Transverse profiles of the third-order correlation coefficient, $\rho^{[1|2]}$, for $M = 1$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

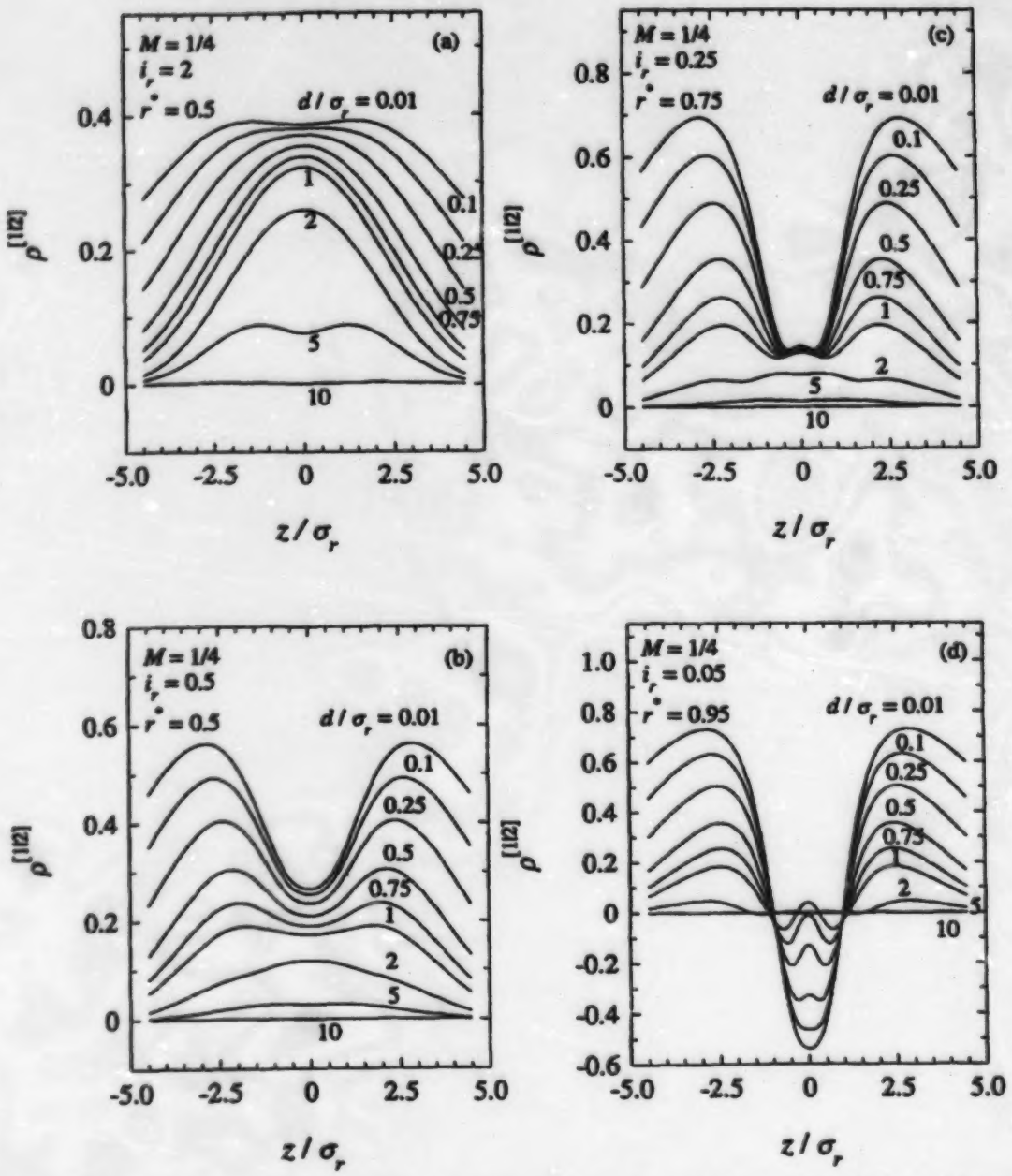

Figure 7. Transverse profiles of the third-order correlation coefficient, $\rho^{[1|2]}$, for $M = 1/4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

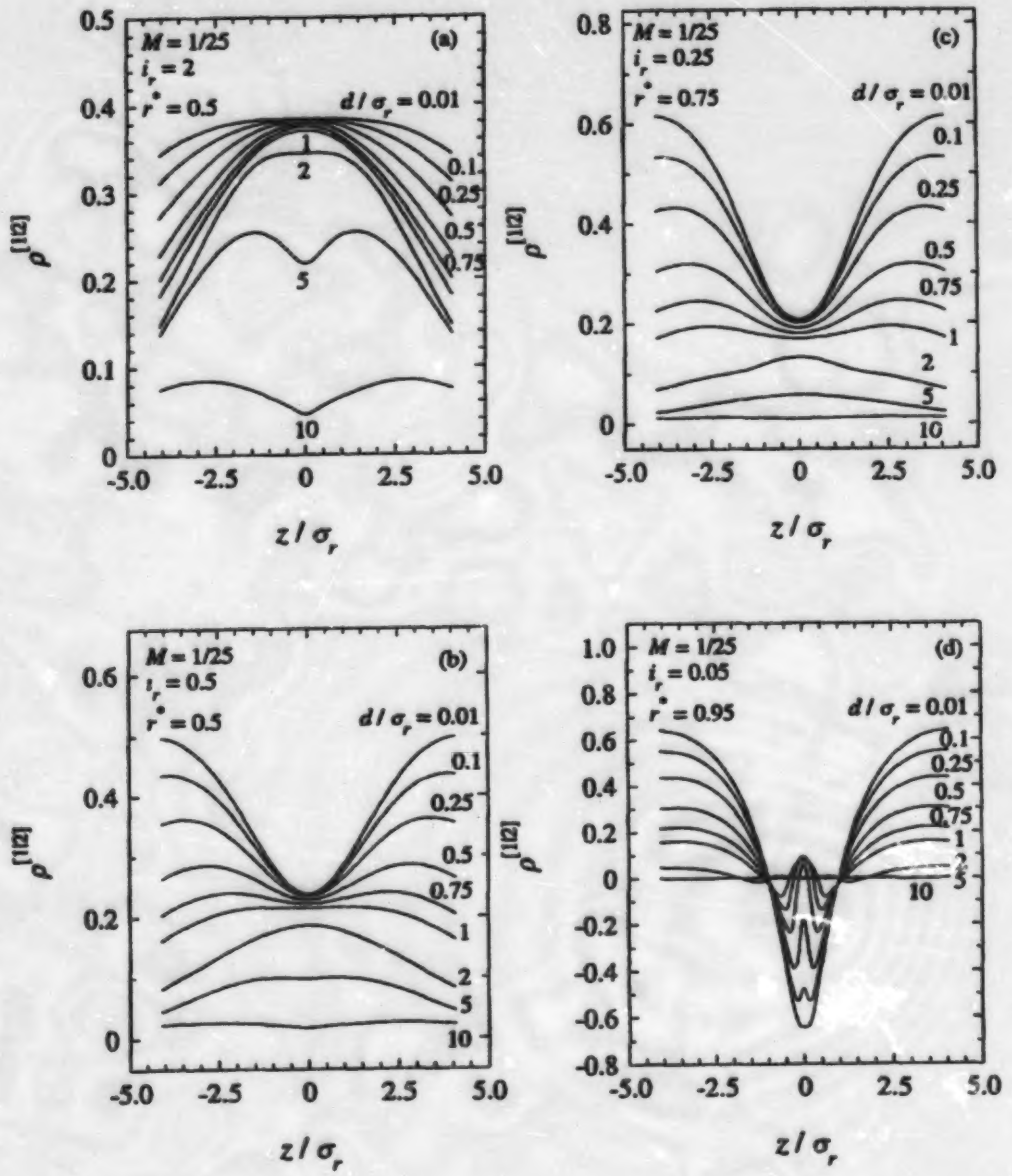

Figure 8. Transverse profiles of the third-order correlation coefficient, $\rho^{[1|2]}$, for $M = 1/25$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

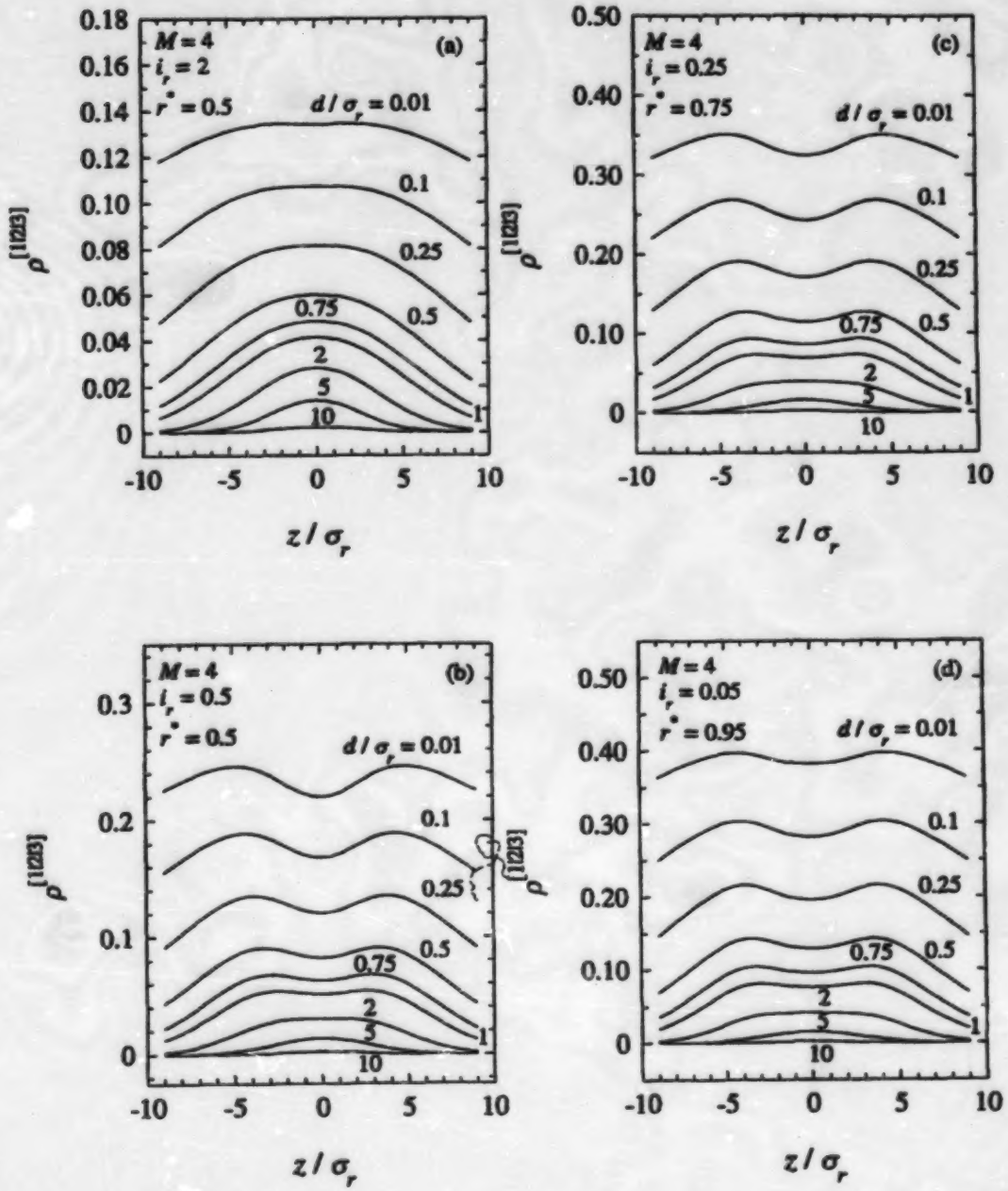

Figure 9. Transverse profiles of the fourth-order correlation coefficient, $\rho^{[1|2|3]}$, for $M = 4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

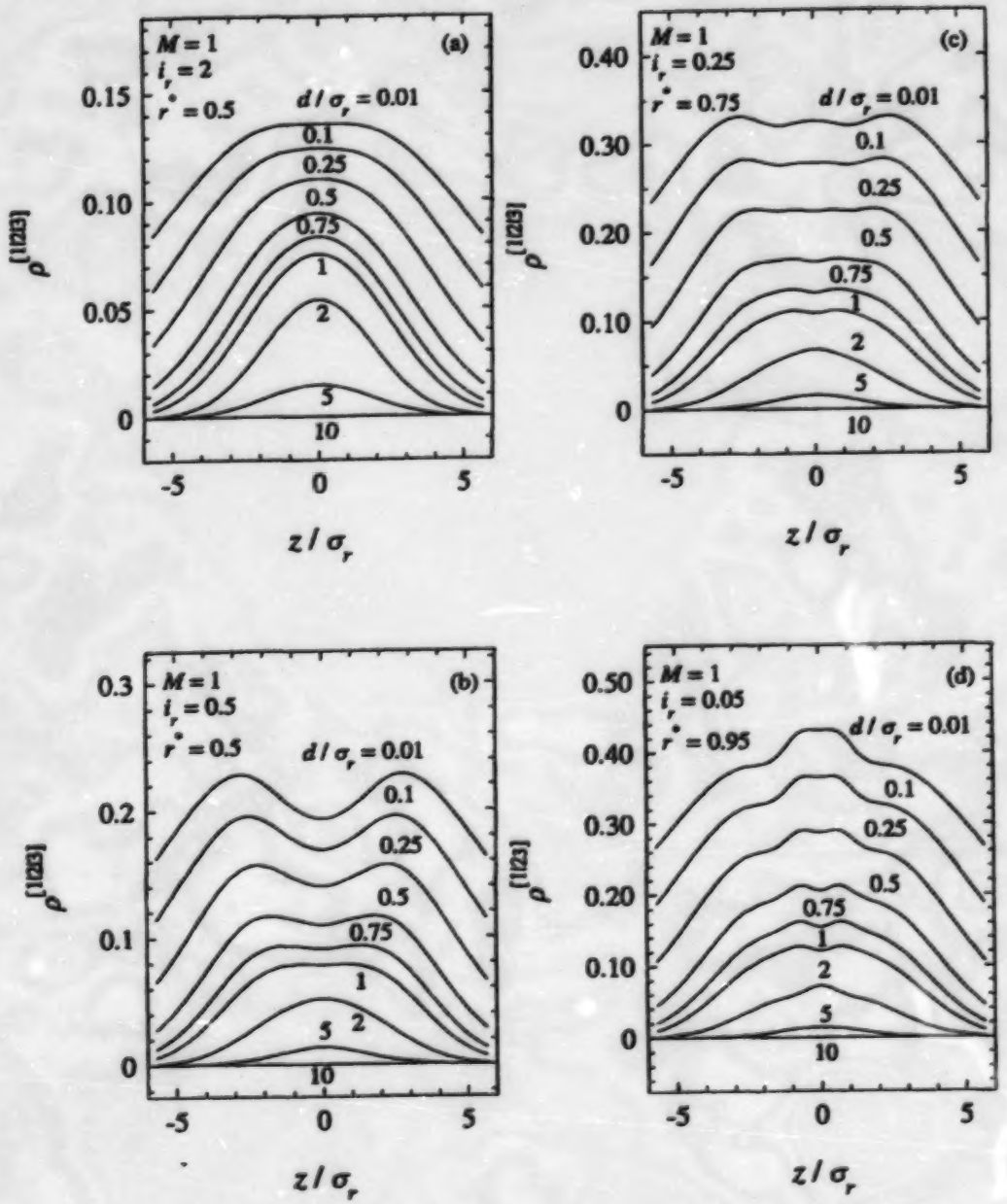

Figure 10. Transverse profiles of the fourth-order correlation coefficient, $\rho^{[1|23]}$, for $M = 1$ and for a range of values of i_r, r°, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

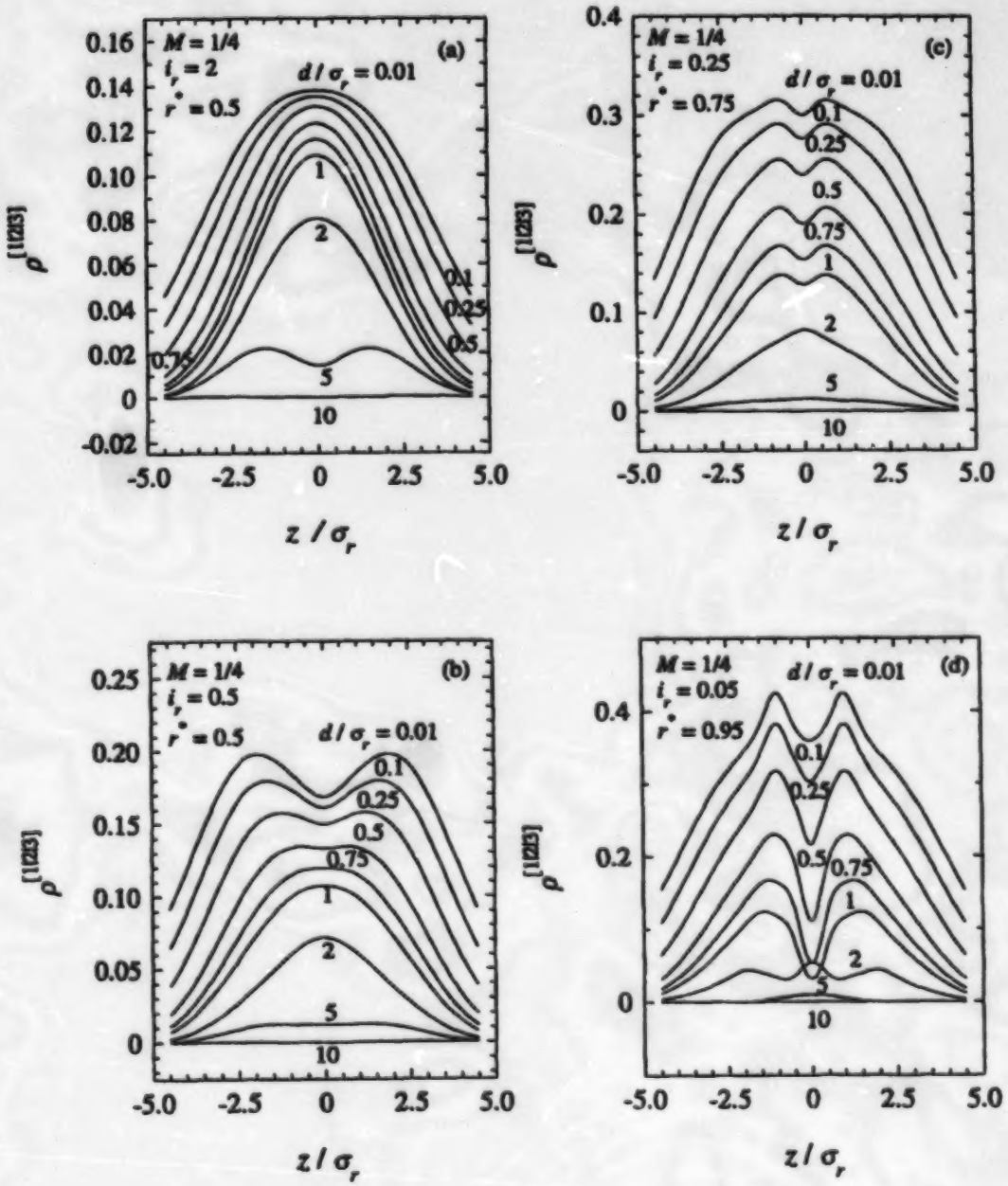

Figure 11. Transverse profiles of the fourth-order correlation coefficient, $\rho^{[1213]}$, for $M = 1/4$ and for a range of values of i_r, r^*, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

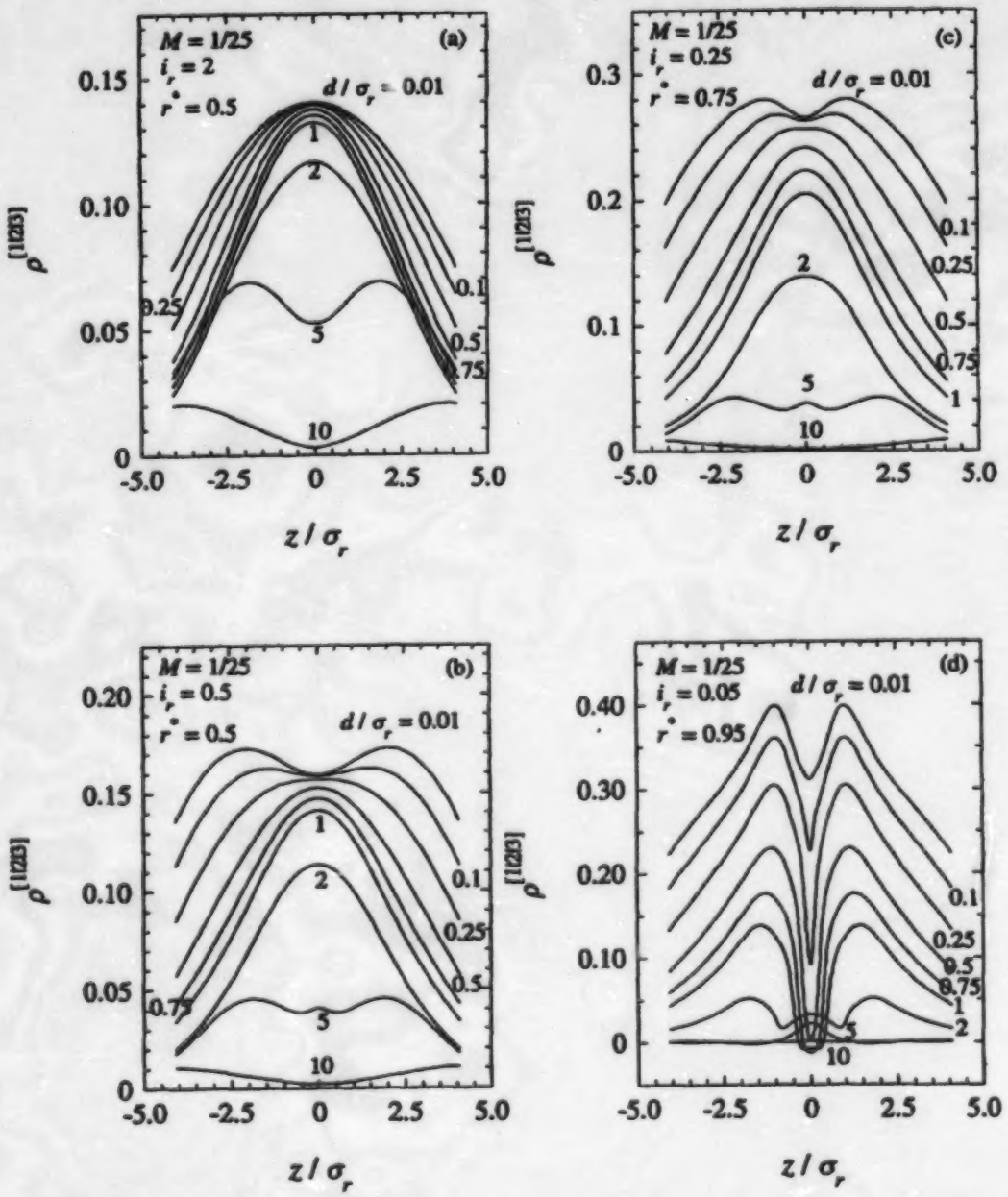

Figure 12. Transverse profiles of the fourth-order correlation coefficient, $\rho^{[1|23]}$, for $M = 1/25$ and for a range of values of i_r, r°, and d. Here, z is the transverse distance in the direction of the source separation referenced to the midpoint between the two point sources.

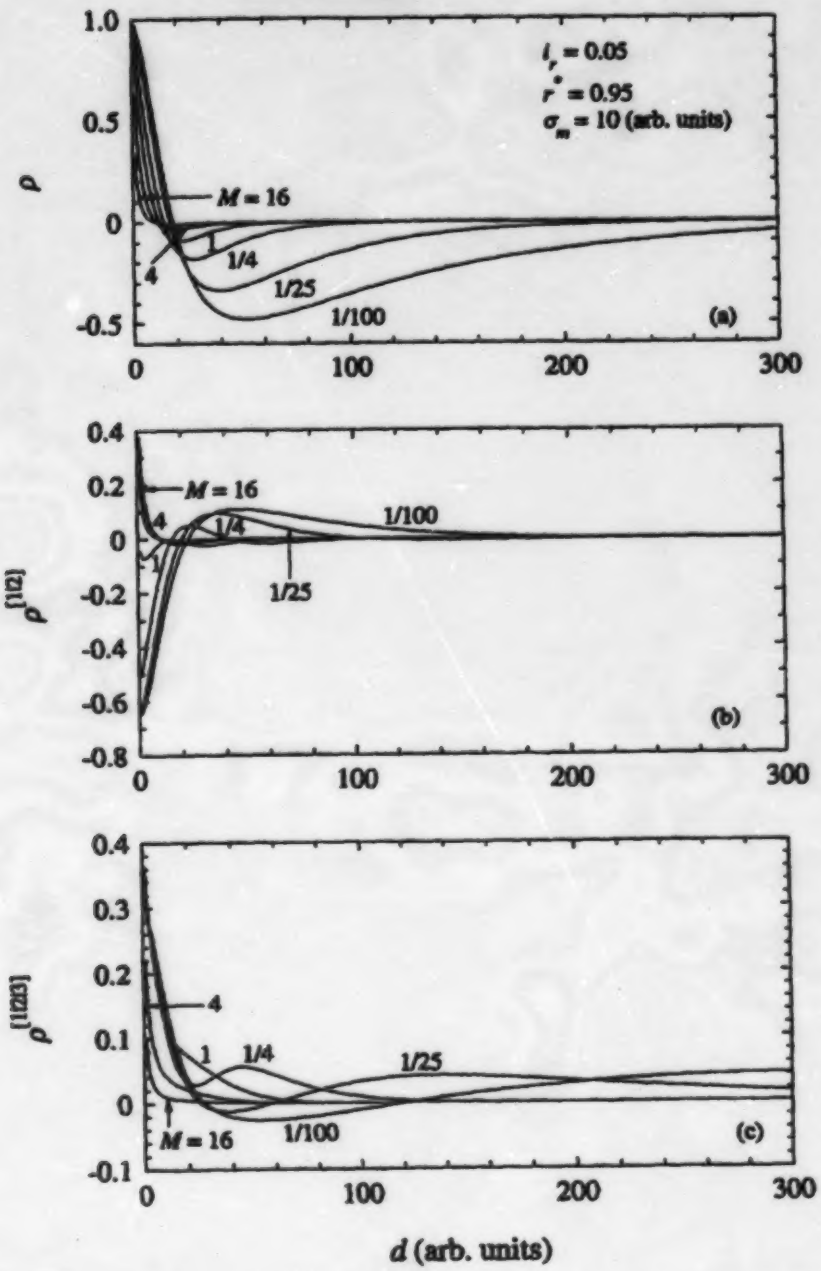

Figure 13. (a) Second-order correlation, ρ; (b) third-order correlation, $\rho^{[1|2]}$; and, (c) fourth-order correlation $\rho^{[1|2|3]}$, between concentrations from two separated point sources as a function of source separation, d, for a range of values of the meander ratio, M.

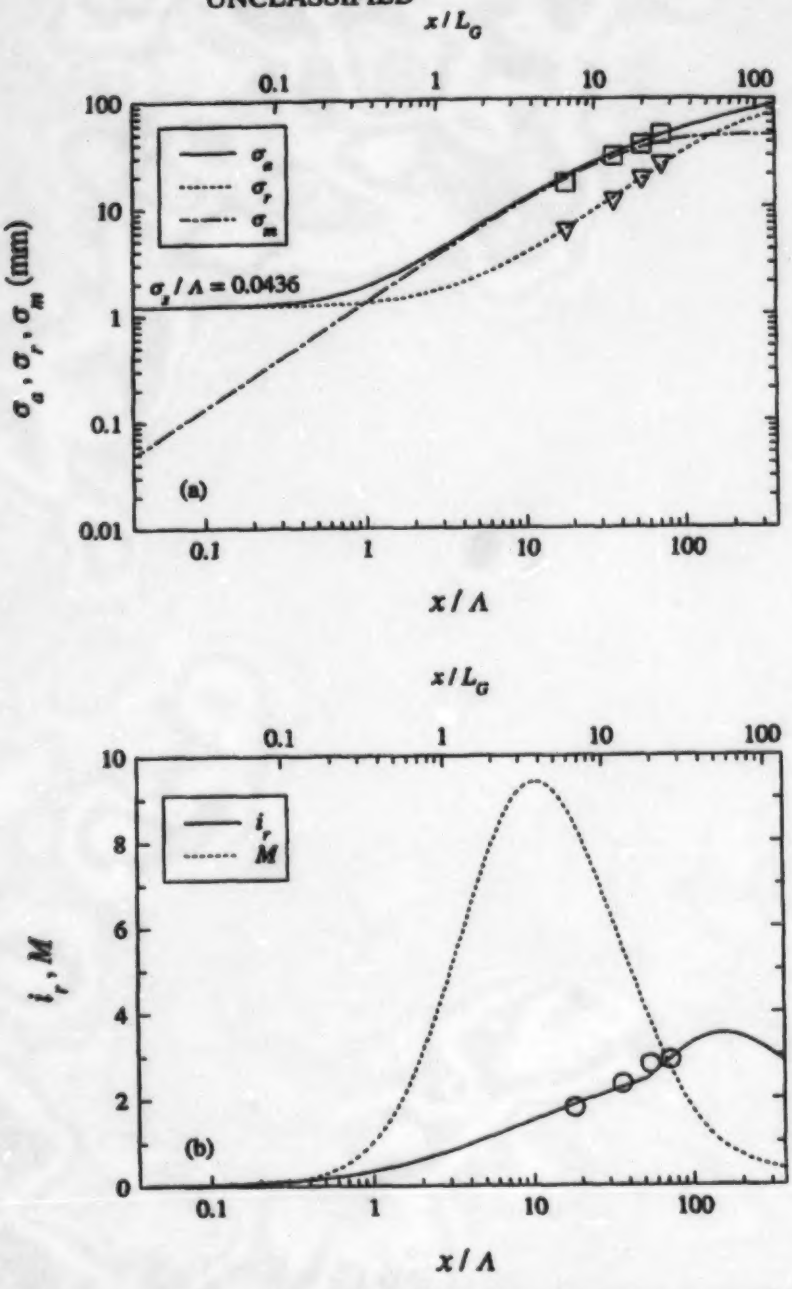

Figure 14. (a) Absolute plume dispersion, σ_a, relative plume dispersion, σ_r, and centroid dispersion, σ_m, calculated as a function of normalized downstream distance, x/Λ. (b) Internal fluctuation intensity, i_r, in the relative frame and meander ratio, M, calculated as a function of normalized downstream distance, x/Λ. The symbols correspond to data obtained from the water channel simulations.

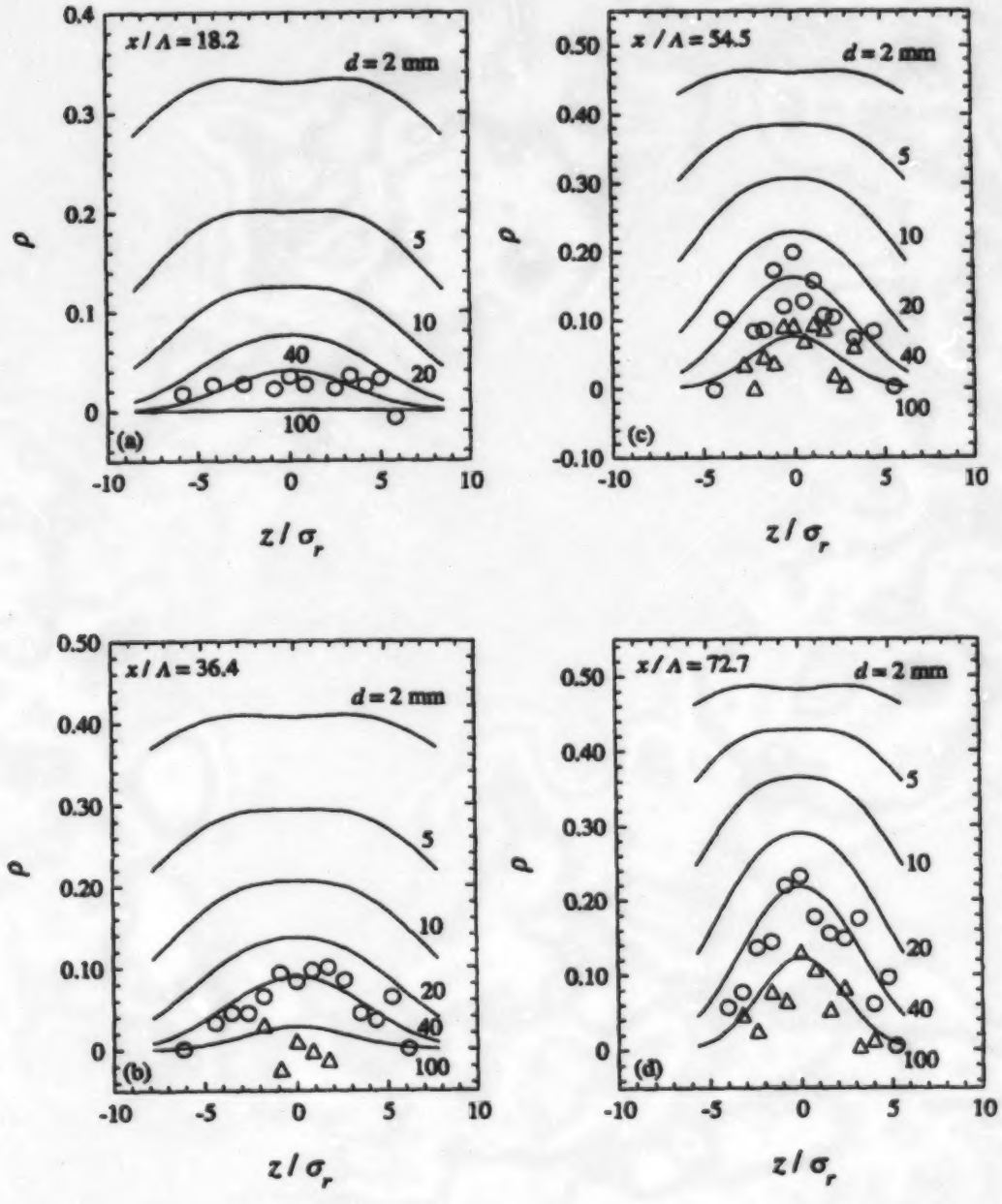

Figure 15. Transverse profiles of second-order correlation, ρ, for a range of source separations at (a) $x/\Lambda = 18.2$; (b) $x/\Lambda = 36.4$; (c) $x/\Lambda = 54.5$; and, (d) $x/\Lambda = 72.7$. Measurements: $d = 40$ mm (O), 100 mm (Δ).

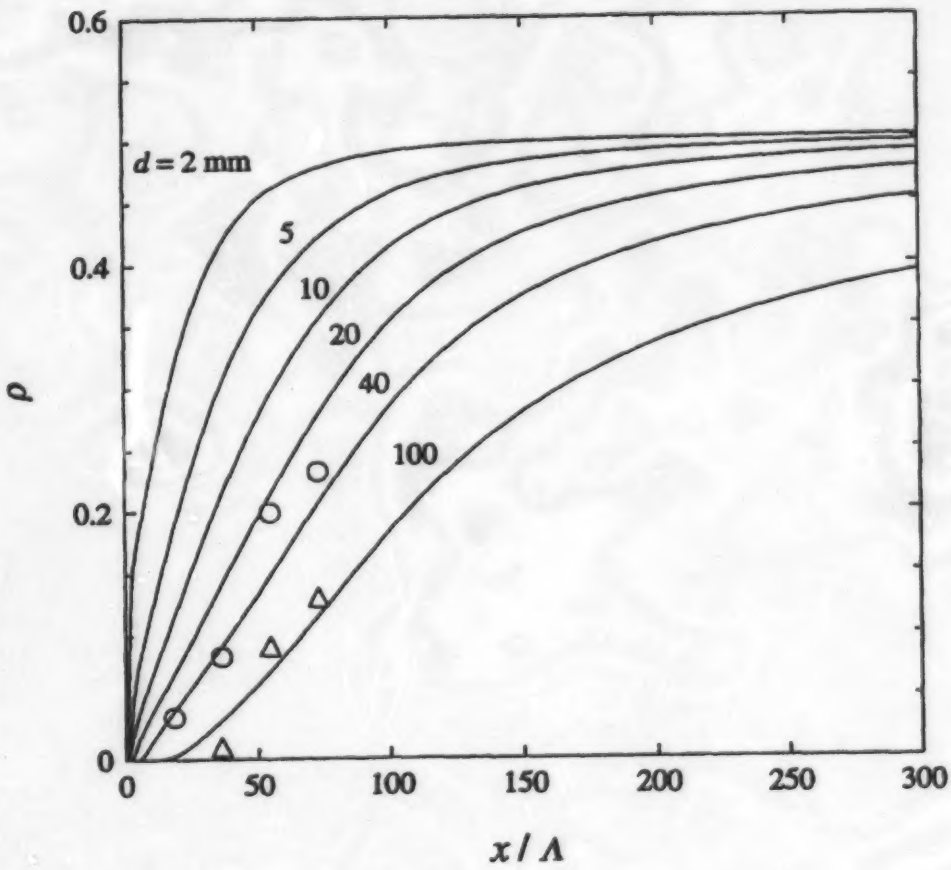

Figure 16. Streamwise evolution of the second-order correlation, ρ, for a range of source separations. Measurements: $d = 40$ mm (O), 100 mm (Δ).

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The higher-order correlation functions for the concentration fluctuations arising from a two-point-source configuration have been calculated analytically within the context of the phenomenology of a fluctuating plume model (viz., a meandering plume model which explicitly incorporates internal fluctuations) Explicit expressions for the second-, third-, and fourth-order correlation functions between the concentration fluctuations produced by two point sources are given in terms of the source separation d and the 5 physically based parameters that define the generalized fluctuating plume model: namely, the absolute plume dispersion, σ_a, which determines the outer plume length scale, the relative plume dispersion, σ_r, which determines the inner plume length scale; the fluctuation intensity, i_r, in relative coordinates which determines the internal concentration fluctuation level; the correlation coefficient, r, between the positions of the centroids of the two interfering plumes; and, the correlation coefficient, r^*, between the concentration fluctuations of the two plumes in relative coordinates which determines the degree of internal mixing of the two scalars. Results for the profiles of the second-order correlation function, ρ, have been compared with some new experimental data for a two-point-source configuration in grid turbulence generated in a water channel simulation. These results are in good agreement with the data and suggests that the analytical model for ρ can reproduce many qualitative trends.

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Dispersion Modelling
Concentration Fluctuations
Interaction of Two Plumes
Higher-order correlation functions
Water channel simulation